

The Inverse Method, Transfer Function Analysis, and Non-Linear Filtering

P.S. Kaiker and G.S. Nusholtz

ABSTRACT

Current computer designs enable tremendously increased computing power. The Fast Fourier Transform economically translates laboratory signals into the frequency domain. Many science and engineering disciplines are beginning to use the **Inverse Method** to determine system characterization parameters from precise laboratory measurements. These developments mean that biomechanists could use the **Inverse Method** and Transfer Function Analysis to determine system characterization parameters for idealizations of human impact response from laboratory measurements. This paper will present the **Inverse Method** as it could be applied to the dynamic biomechanics of thoracic trauma, illustrating, in particular, some of the signal processing required in order to use mechanical impedance data to develop models of thoracic impact response.

INTRODUCTION: THE INVERSE METHOD

Increased computing power and the ability to make highly accurate measurements is altering the type of problems posed by scientists and engineers. For example, a thermal engineer, knowing the thermal conductivity of a material, may not be particularly interested in ways of predicting the temperature at a point in that material body as a function of time because such a variable can be accurately measured. However, if the thermal conductivity of a composite material is unknown or difficult to predict, the engineer can determine the thermal conductivity of the composite material from its temperature profile by extrapolating. System characterization problems of this type are "inverse" problems. Generally speaking, "inverse" problems are material property determination problems. Such properties include density, mass, conductivity, resistance, and elastic constants, for example. The determination of material properties is readily derivable from the system being studied. For example, from an **Inverse Method** analysis of the pattern of seismic waves reflected and refracted by the layers of rock through which the waves had passed, information can be obtained about the density and the nature of the rock. This is an example of an inverse scattering problem. When the **Inverse Method** is used to analyze the health of an arterial wall, a weakened region of the arterial wall is detected by spectral analysis of the ultrasonic pulse data. The **Inverse Method** is also used to study macromolecules, such as proteins and DNA. X-ray crystallographs provide data from which the configuration as well as the constituents of the macromolecules are determined. The **Inverse Method** is used to identify faults within the not-easily accessible heart of a nuclear reactor from neutron and meson probe data.

Trauma biomechanists could pose similar inverse problems. For example, assume that a material body, such as the thorax, is being modeled as moving along one dimension. The impact behavior of the thorax, expressed as a function of time, could be characterized by the following equation:

$$F(t) = M * A(t) \quad (1)$$

where M equals the mass of the thorax, F equals the applied force, and A equals the measured thoracic output, acceleration. Using the "Direct Method," knowing the mass and the force, the acceleration would be predicted. However, there are times when the mass of a system is not well-defined or determinable. The **Inverse Method** seems useful for such difficulties. Using the **Inverse Method**, knowing force, acceleration would be measured, and **effective mass** would be predicted. For example, the effective values of thoracic material properties for idealizations of thoracic impact response could be derived from precise laboratory measurements.

DETERMINATION OF THORACIC MATERIAL PROPERTIES BY INVERSE METHOD TRANSFER FUNCTION ANALYSIS

The **Inverse Method** determination of the material properties of the thorax undergoing blunt impact may consist of a number of stages. The first stage is usually the selection of a set of models or idealizations of the system's impact response. For example, the equation mentioned earlier:

$$F(t) = M * A(t) \quad (1)$$

is an example of one model of thoracic impact response. The first stage of the **Inverse Method** often incorporates a review of the information available on the impact response of the anatomical system. This review might suggest that the impact behavior of the thorax would be most economically describable by a set of idealizations or by a particular simulation model, such as a lumped-parameter, finite element, or continuous media model. A number of models might seem equally plausible and worthy of investigation during this first stage.

The next stage is the theoretical design of laboratory experiments. Can the needed parameter be uniquely determined from the variables measured? In other words, given input to the system and output from the system will the parameter required be uniquely determined? That is, given the force input to the thorax and measuring the thoracic output (e.g., skeletal accelerations, vascular, and pulmonary pressures), will the effective thoracic mass be uniquely determined? At this stage, the effect of errors of measurement, sample size, and sampling interval on the statistical estimation of parameter values is considered. The experimental design should minimize the variance of estimates of parameter values as much as possible.

The following stage is the design of the laboratory protocol for the impact testing and includes performing the experiments. The laboratory protocol not only represents the formal presentation of the hypotheses being investigated, but also outlines all of the surgical, mechanical, and electromechanical procedures used to control the quality of the transducer time-histories. These procedures include surgical ones for rigidly affixing transducers to the thorax, mechanical procedures for increasing the signal-to-noise ratio (e.g. the design of

triaxial accelerometer mounts and mounting platforms), and procedures for adjusting amplifiers and tape-recorders to optimum ranges for data-gathering.

Trauma producing blunt non-penetrating impacts to human surrogates have been studied for several years. These experiments are re-creations of the impact environments, occurring as a result of a sports, industrial, military, or automotive accident. Much of the experimental work has utilized the repressurized human cadaver as the impact test subject, however experimental animals have been studied when basic physiological responses were being investigated. The test subject is sanitarily prepared and surgically instrumented prior to testing with mounting platforms for the measuring devices. The major testing equipment usually includes: 1) an impact device for delivering a calibrated amount of energy to the thorax, 2) a timing control unit to synchronize the electrical equipment for a 10-100 ms impact, 3) a high-speed x-ray cineradiograph or cameras and lights to photographically record gross motion of the thorax, 4) electromechanical accelerometers, pressure transducers, strain gauges, and associated amplifiers to measure the kinematic response for a given impact condition, 5) test subject repressurization equipment, 6) FM data tape machines to record the electromechanical transducer time-histories, 7) support materials or supporting systems to pre-position the test subject in a precise postural configuration prior to impact, and 8) pathological equipment, digitizers, and computers to process the data before its final analysis.

The data-processing stage includes a post-test detailed autopsy or necropsy to record the damages resulting from the impact, the digitizing of the high-speed films, and the processing of the transducer time-histories. The transducer time-histories are processed with a non-linear shift-variant filter to increase the signal-to-noise ratio in order to provide optimum conditions for transfer function analysis.

During the analytical stage, analysis of the pathological and photogrammetric data plus the electromechanical signals provides a framework in which kinematic motion parameters such as forces, velocities, accelerations and rotations can be used to estimate the **material properties of the impacted thorax** in terms of both the time and frequency domains. The inverse procedure for obtaining material properties from laboratory signals can use many frequency domain transfer functions. **Mechanical Impedance** is only one of these. First, the filtered transducer time-histories are transferred from the time domain using the Fast Fourier Transform into the frequency domain. Then mechanical impedance transfer functions are generated from the force at a given point in the system and the velocity at any other given point. A thoracic material property such as effective mass is determined inversely from the mechanical impedance data.

Mechanical Impedance - Mechanical impedance techniques can facilitate the understanding of an anatomical system's kinematic response to blunt impact. For a mechanical system excited by a sinusoidal force input of a given frequency, as is the case during blunt impact to the thorax, it might be desired to determine the velocity of a remote point in the system that is of the same frequency. The quantity relating the force and resulting velocity of the same frequency is called **Mechanical Transfer Impedance (Z)**. Force and acceleration transducer time-histories are monitored during the impact and a Fast

Fourier Transformation (FFT) is performed on the digitized signals. From the digitized data, a transfer function of the form:

$$Z(i\omega) = \omega F[F(t)]/F[A(t)] \quad (2)$$

is generated, where ω is the given frequency and $F[F(t)]$ and $F[A(t)]$ are the Fourier Transforms of the impact force and acceleration, respectively, of the point of interest.

Based on mechanical impedance analysis of a single impact, a model of the thorax for that particular impact can be generated in terms of physical parameters. The model might be generalized to fit a number of similar systems, if either sufficient impact data were obtained or sufficient properties about the physical system were known. Such a model assumes that the impact response of the human thorax might be approximated as second-order linear. A second-order linear mechanical system is describable in terms of physical parameters such as springs, masses, and dampers.

The general equation of motion for a mechanical system containing a spring, mass, and damper in parallel can be written in terms of velocity as:

$$m \, dv(t)/dt + cv(t) + k \int v(t) \, dt = F(t) \quad (3)$$

where v = velocity

m = system apparent mass

c = damping ratio

k = spring constant

F = time-dependent forcing function.

Alternatively, it can be expressed in terms of force as:

$$1/k \, dF(t)/dt + 1/c \, F(t) + 1/m \int F(t) \, dt = v(t) \quad (4)$$

These equations are analogous to those which may be written for an electrical RLC circuit, with respect to Kirchoff's current law as:

$$l \, di(t)/dt + Ri(t) + 1/c \int i(t) \, dt = e(t) \quad (5)$$

and with respect to Kirchoff's voltage law as:

$$c \, de(t)/dt + 1/R \, e(t) + 1/l \int e(t) \, dt = i(t) \quad (6)$$

where i = current

c = capacitance

R = resistance

l = inductance

$e(t)$ = time-dependent voltage input.

The mechanical and electrical systems are compared as follows:

MECHANICAL SYSTEM QUANTITY	ELECTRICAL SYSTEM ANALOG
Velocity	Current
Force	Voltage
Mass	Inductor
Damper	Resistor
Spring	Capacitor.

The procedure of Fourier Transformation for solving the differential equations describing an electrical system, therefore, can be applied to the analogous mechanical system. The procedure involves transforming the differential equation of motion into the complex frequency domain to obtain an algebraic equation which can be solved much more easily. This procedure assumes that the system is time invariant, linear, and that the principle of superposition can be applied. In addition, it is assumed that the initial conditions of the system are all zero and that the magnitude of the response at any given frequency is a result of an excitation of the same frequency. Letting the variable x denote the displacement of the system mass from rest, equation (3) can be written as:

$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (7)$$

Fourier Transformations to the frequency domain of $x(t)$ and its time derivatives are as follows:

$$\begin{aligned} F[x(t)] &= X(\omega) \\ F[\dot{x}(t)] &= \omega X(\omega) \\ F[\ddot{x}(t)] &= \omega^2 X(\omega) \end{aligned} \quad (8)$$

Performing this operation on equation (7) results in:

$$F(m\ddot{x} + c\dot{x} + kx) = F[F(t)] \quad (9)$$

and since the operation is linear and m , c , and k constant:

$$mF(\ddot{x}) + cF(\dot{x}) + kF(x) = F[F(t)] \quad (10)$$

Therefore by the relationships of transformations (8):

$$\begin{aligned} m\omega^2 + c\omega X + kX &= F(\omega) \\ (m\omega^2 + c\omega + k) X(\omega) &= F(\omega) \\ (m\omega + c + k/\omega) X(\omega) &= F(\omega)/\omega \end{aligned} \quad (11)$$

From the form of equation (11), it is apparent that a quantity increasing linearly with the frequency ω is behaving as a mass; a quantity constant with ω is behaving as a damper;

and a quantity decreasing linearly with ω is behaving as a spring. The mechanical impedance Z of a system is defined as the complex ratio of the Fourier Transform of the force to the Fourier Transform of the velocity:

$$Z(i\omega) = F[F(t)]/F[V(t)] \quad (12)$$

where the magnitude of the complex-valued quantity Z is increasing linearly with frequency ω , the system is behaving as a mass (Figure 1a); where Z is constant with ω , the system is behaving as a damper (Figure 1b); and where Z decreases linearly with ω , the system is behaving as a spring (Figure 1c). This is apparent from the form of equation (11). Figures 1d, 1e, and 1f show the impedance response of various combinations of these elements. For example, in Figure 1f, given a low frequency input, the system responds as a rigid body with a characteristic mass. However, when the frequency content of the input signal increases, energy is dissipated in the damper, and the mass is not displaced. Thus, the mechanical impedance plots of the impact response of the thoracic system can be analyzed in this manner.

During the final Inverse Method stages, the analyzed mechanical impedance information can be used for modelling the impact response of the thorax. The mechanical impedance information is used to suggest a model. The model is represented as a transfer function(s) in the frequency domain. The model transfer function(s) then is adjusted to fit the laboratory transfer function(s) in the least-squares sense. An inverse transform of the idealization predictions into the time domain enables a comparison of the model's predictions to laboratory data which were not initially used to determine the model's parameters.

MECHANICAL IMPEDANCE CONTINUOUS MEDIA MODEL

All mechanical systems have distributed mass. If the deformation of the system under study is small, or the vibration frequency associated with the input is low enough so that the time of travel of a stress wave is little compared to its period, then the mass can be considered as a lumped element. However, in the case of anatomical structures this condition, in general, does not occur. Therefore, it is reasonable to assume that some form of distributed mass should be included in an idealization used to characterize the impact response of the thorax. Just as the equations for a discrete lumped parameter model can be written in terms of ordinary differential equations, the equation for a linear distributed mass model can be written in terms of partial differential equations:

$$G = \frac{\partial^2 \psi}{\partial X^2} + \mu \frac{\partial^3 \psi}{\partial t \partial X^2} - \rho \frac{\partial^2 \psi}{\partial t^2} \quad (13)$$

where ψ is the field variable under study, X is the coordinate, G is the elastic modulus, μ is the viscous coefficient, and ρ is the density of the material. This equation (or its slight modification) has been used, for example, to characterize a long thin rod, a thin disk in tension or compression, a shear sandwich, a helical spring, a bar in bending, and a block of rubber. Just as ordinary differential equations can be transformed into algebraic equations in

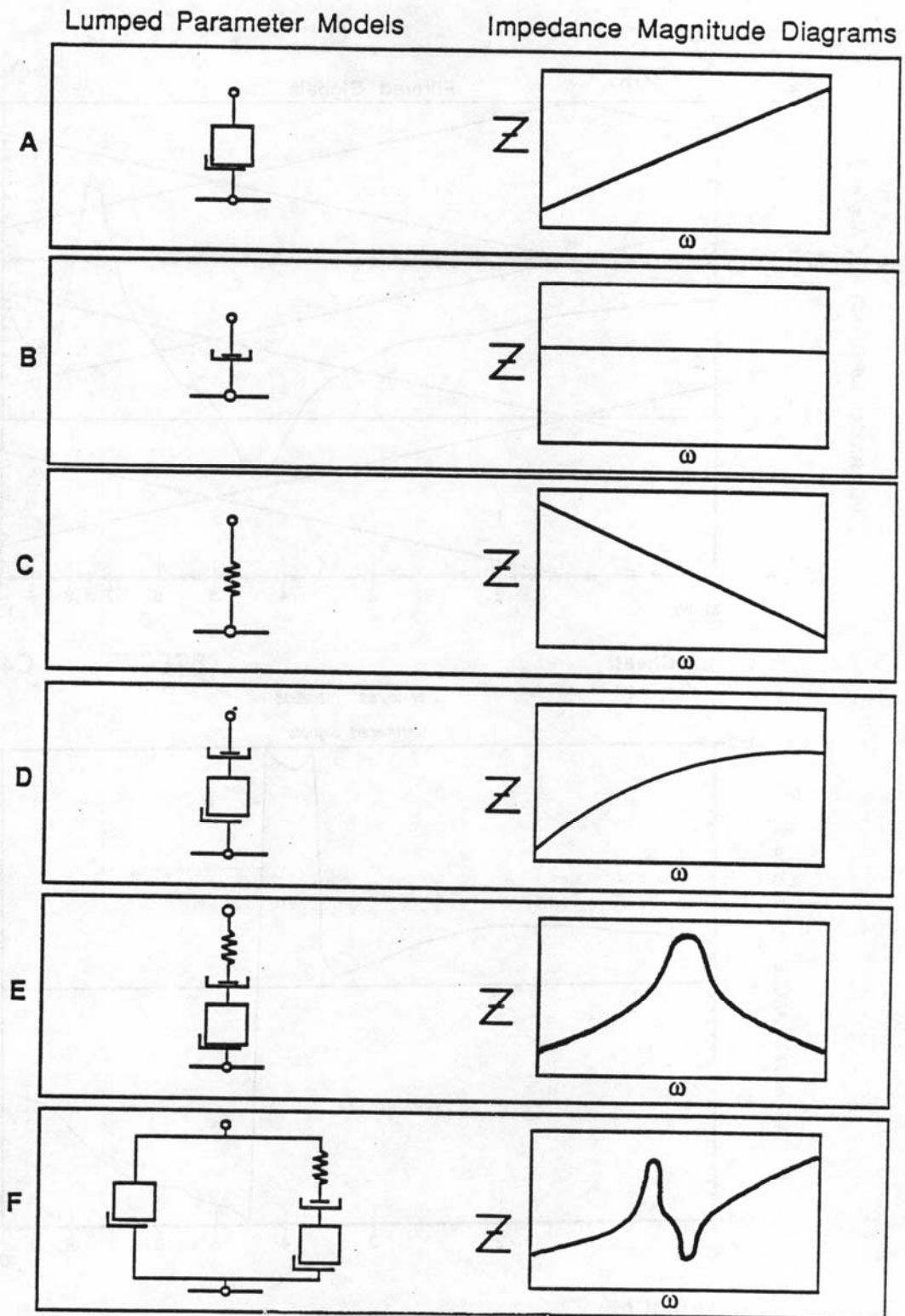


Figure 1. Comparison of Lumped Parameter Models and Impedance Magnitude Diagrams

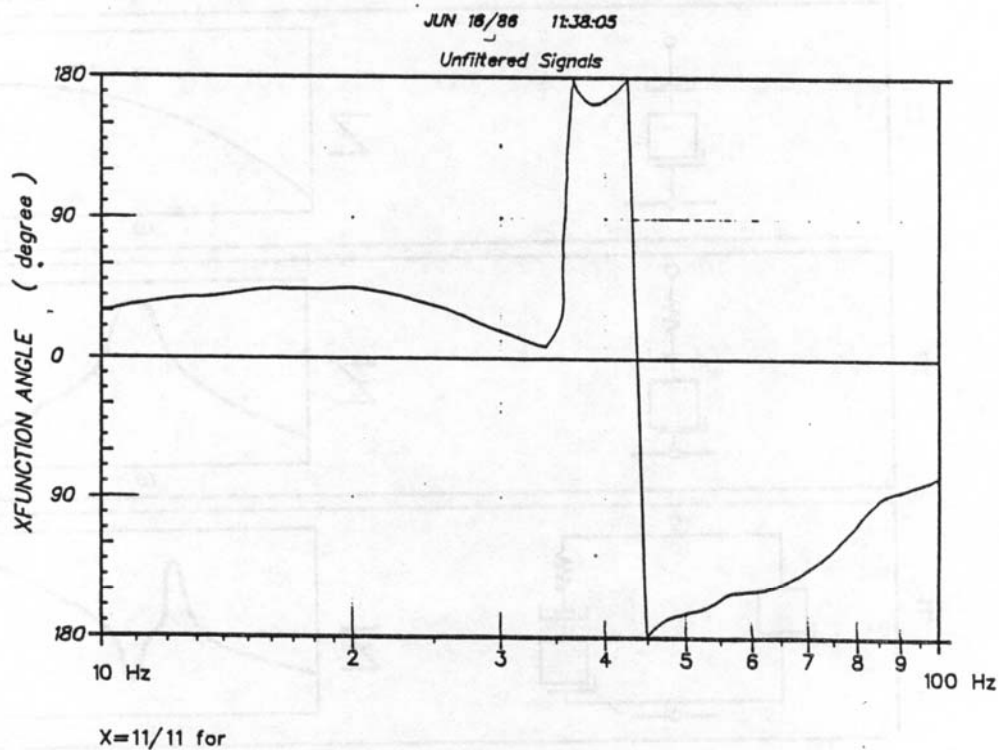
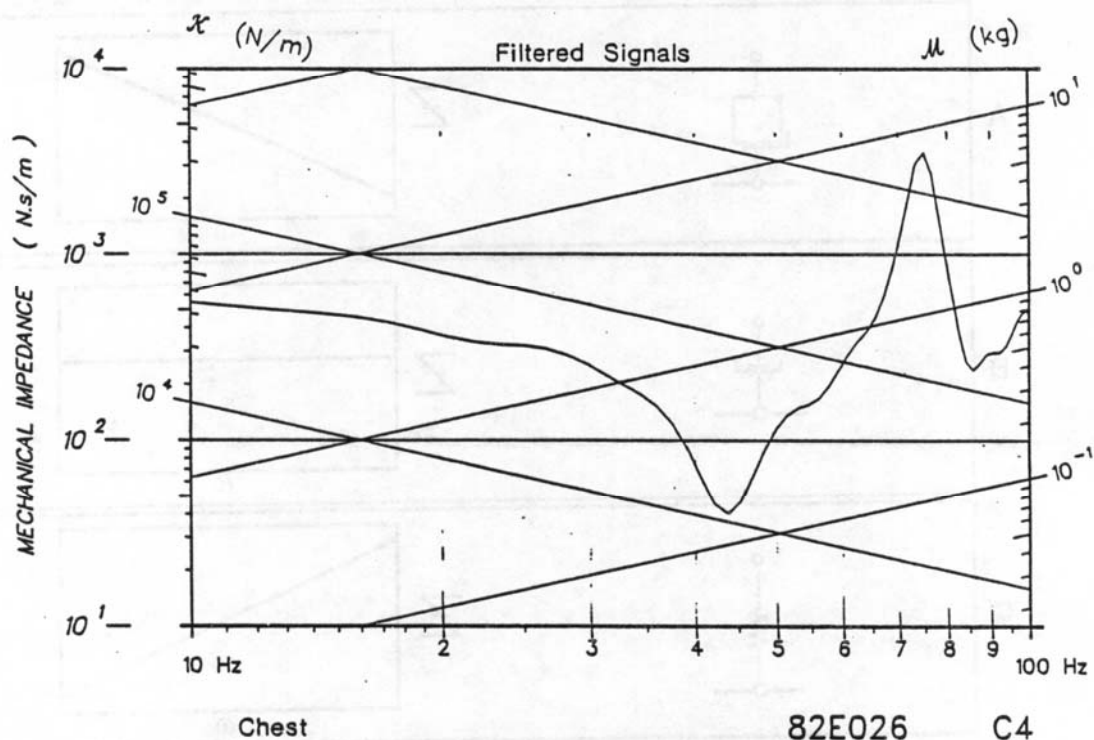


Figure 2. The Magnitude and Phase Angle of the Mechanical Impedance Transfer Function

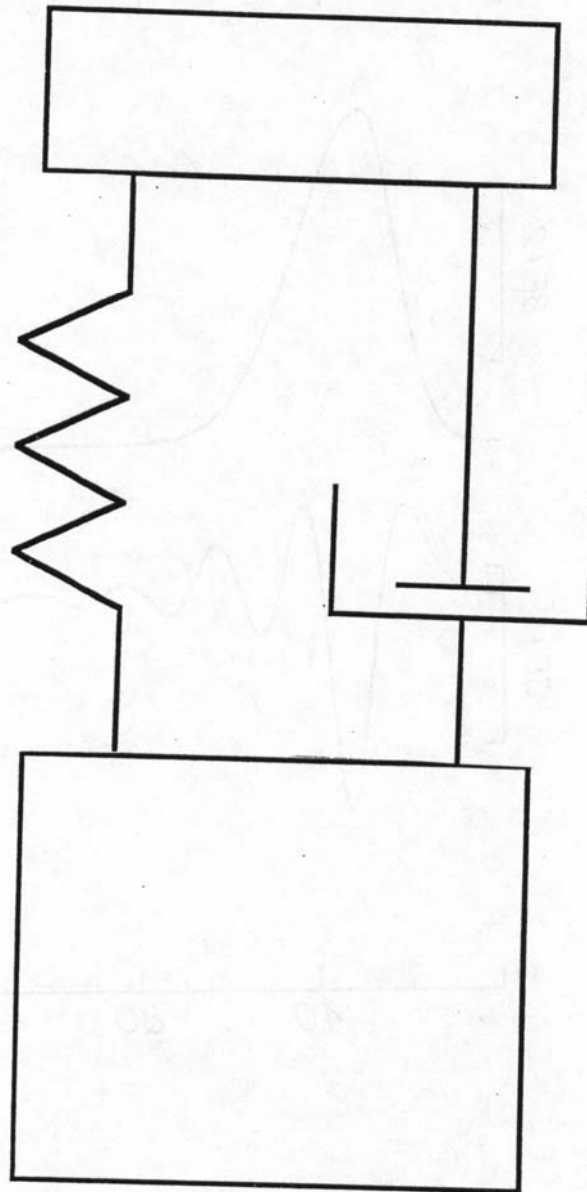


Figure 3. A Two-Mass Model Based on Only the Mechanical Impedance Magnitude

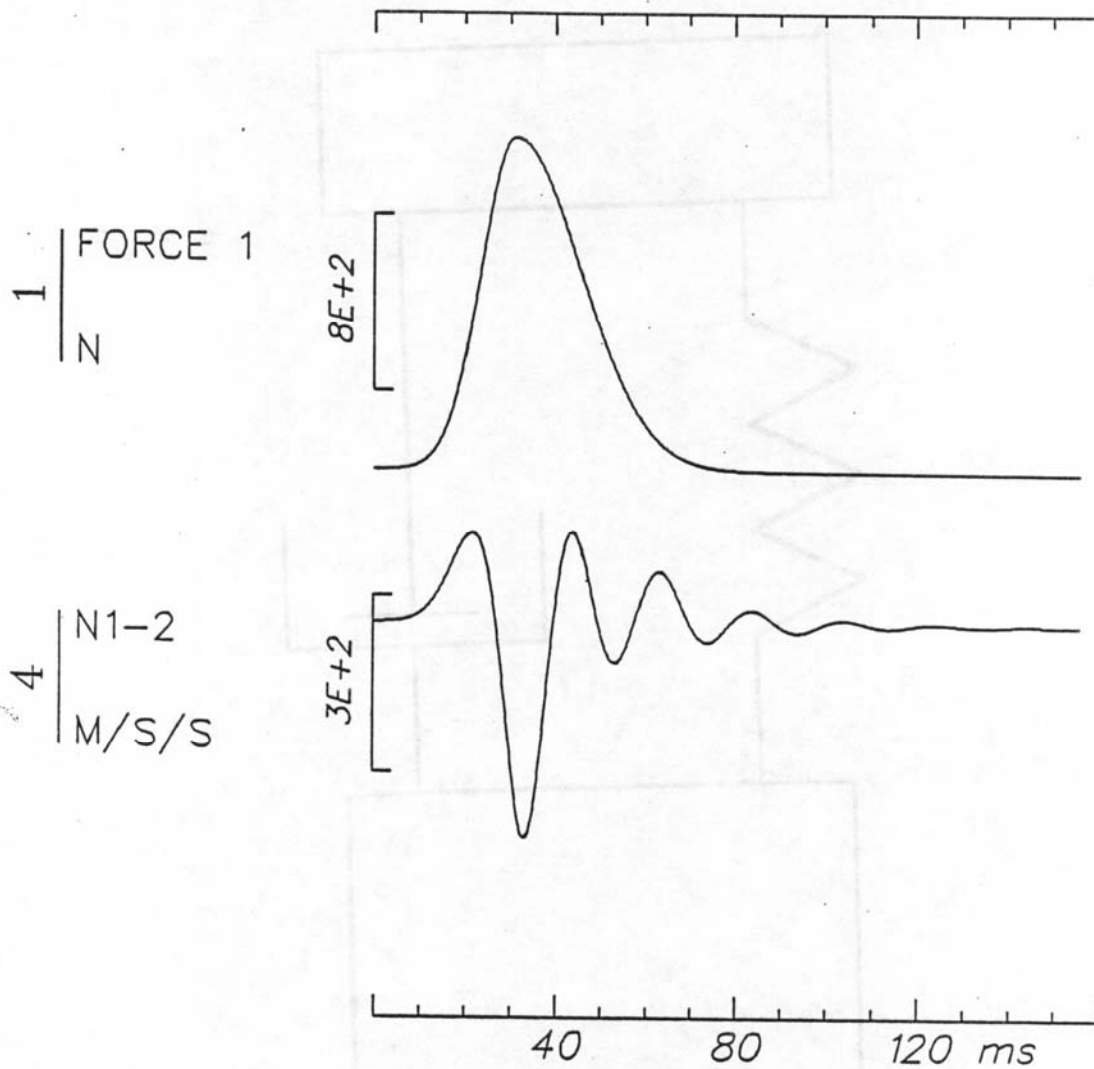


Figure 4. A Two-Mass Model Based on Only the Mechanical Impedance Magnitude Does not Accurately Reproduce the Acceleration Time-History

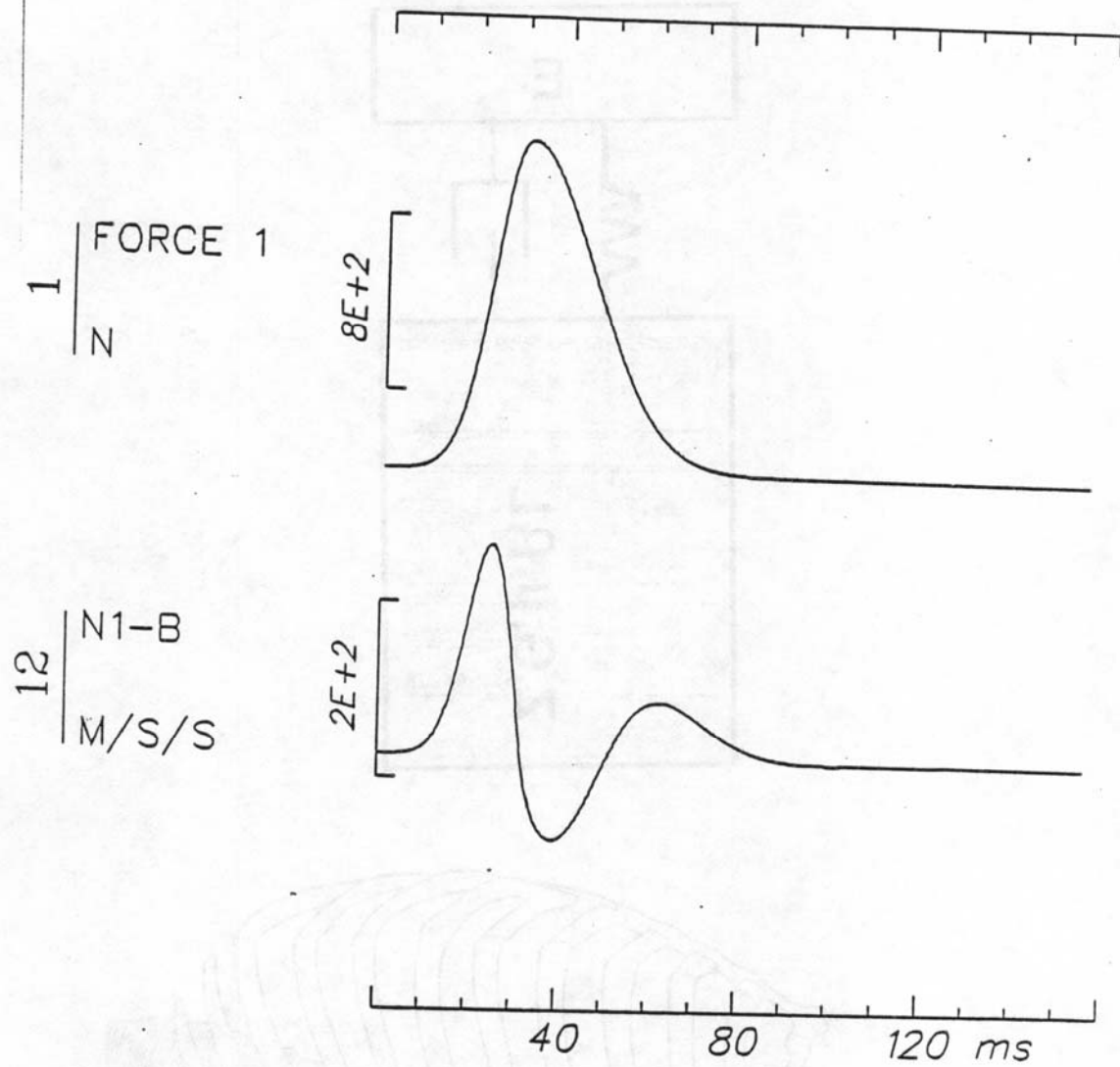


Figure 5. The Result of a Two-Mass Model Based on Both the Mechanical Impedance Magnitude and Phase Angle

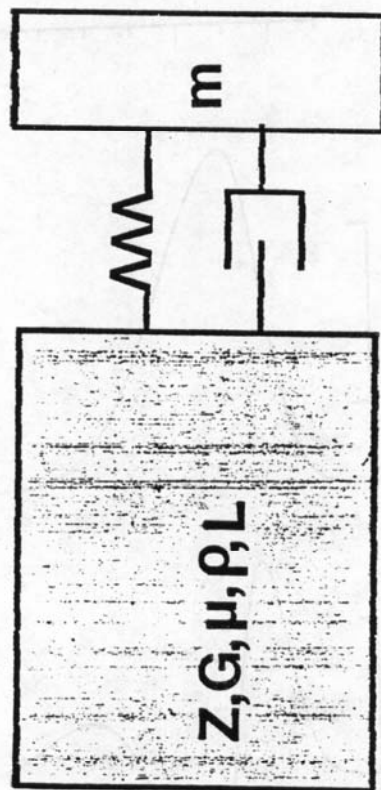
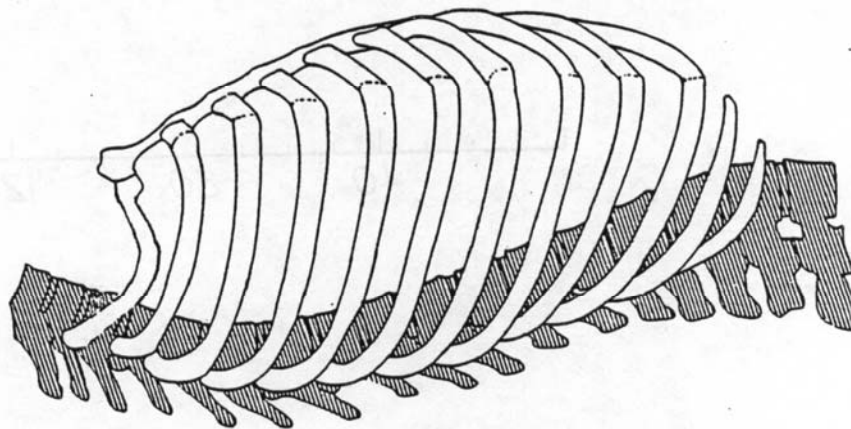


Figure 6. A Simple Continuous Media Model of the Thorax

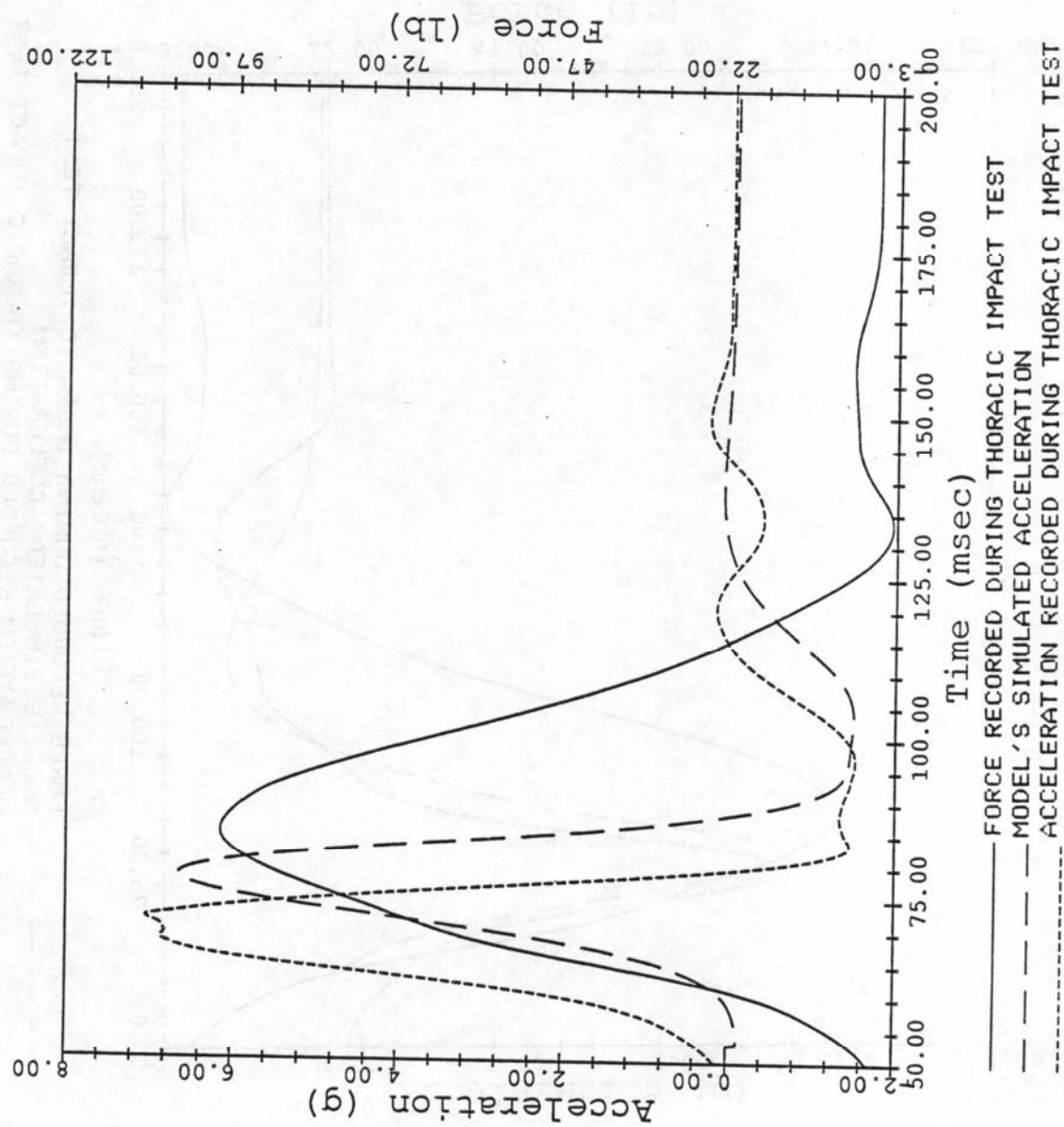


Figure 7. Comparison of an Acceleration Time-History of a Sternum and the Force Input Plus the Model's Predicted Acceleration

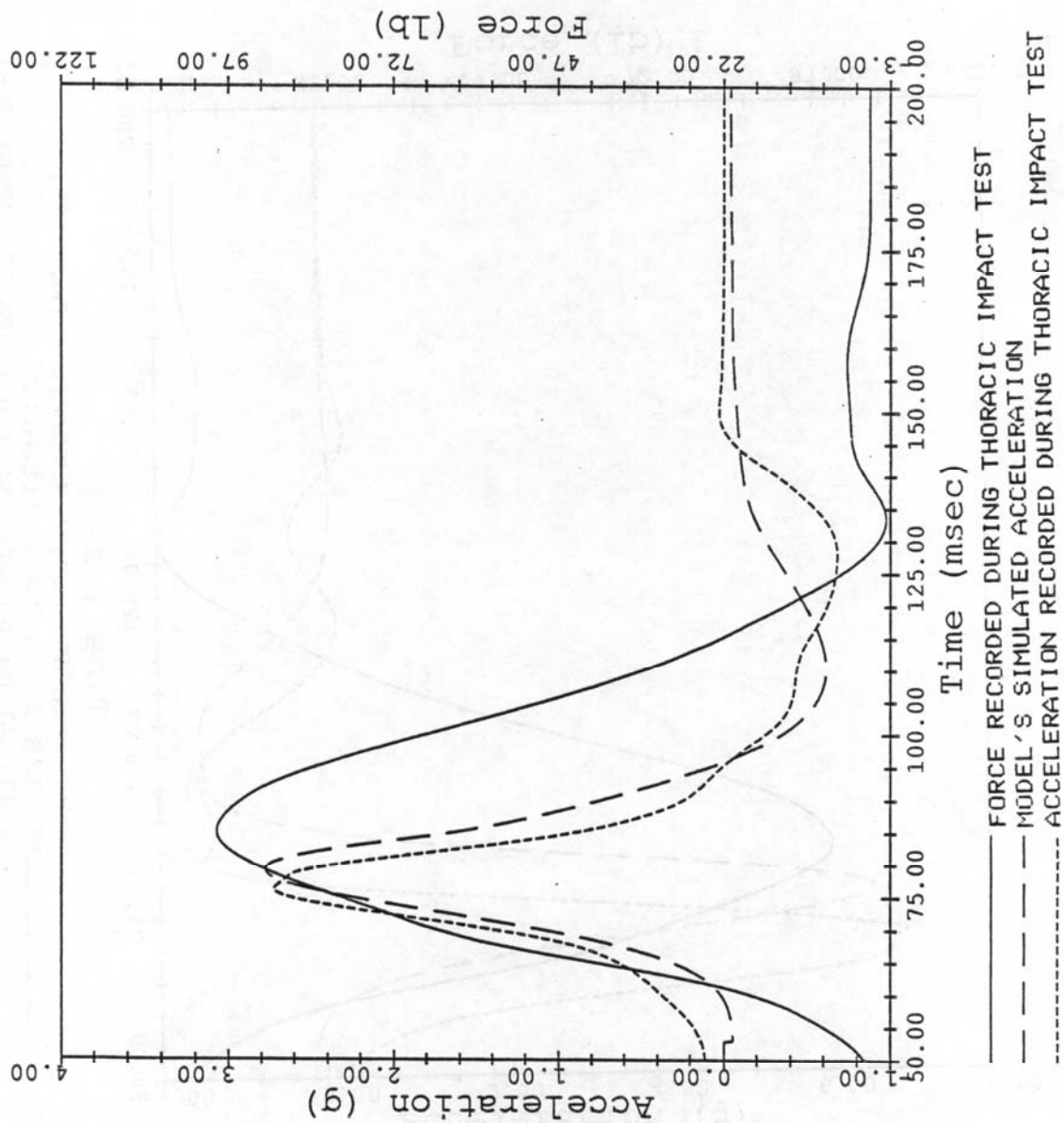


Figure 8. Comparison of an Acceleration Time-History of a Right Fourth Rib and the Force Input Plus the Model's Predicted Acceleration

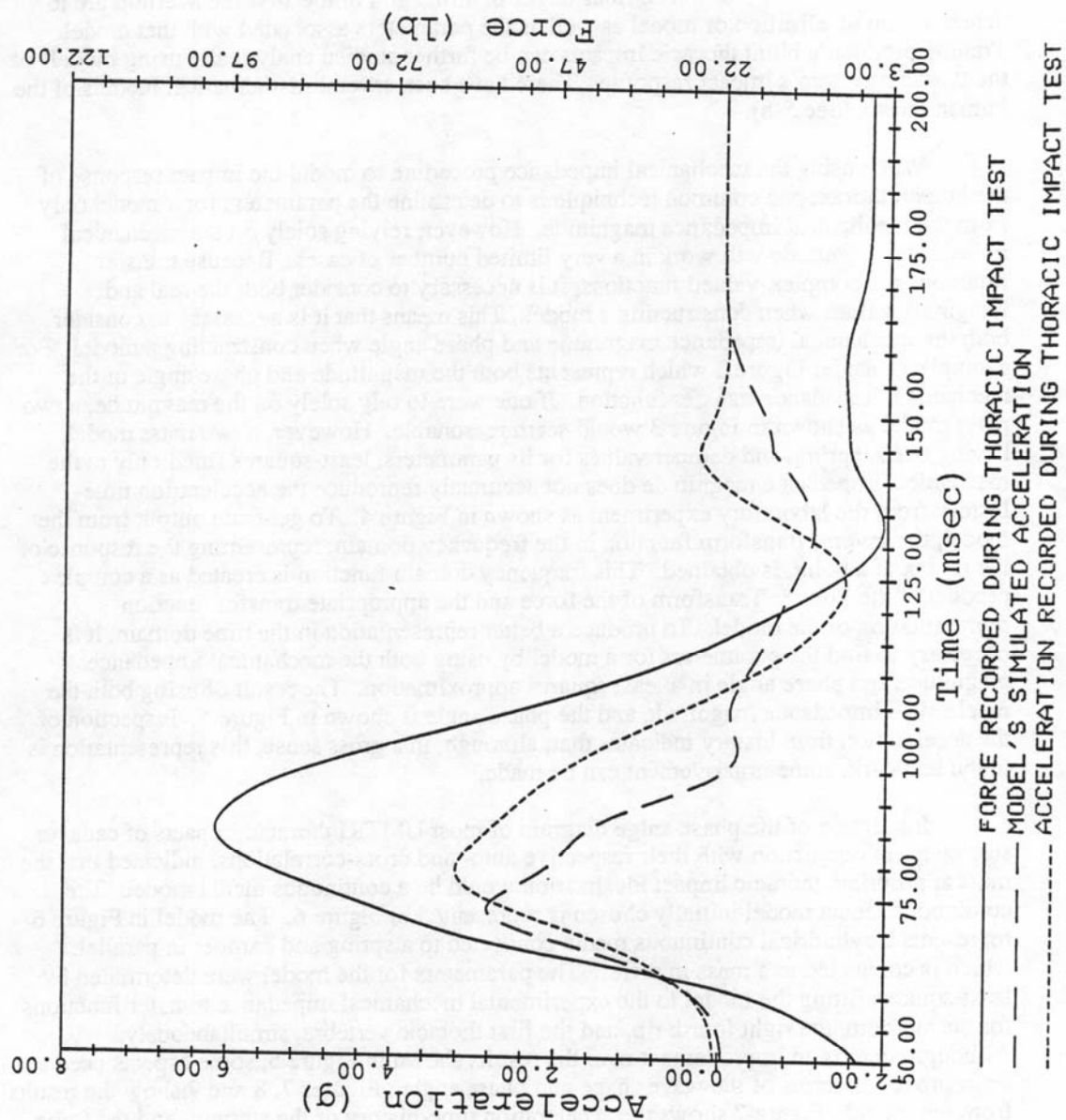


Figure 9. Comparison of an Acceleration Time-History of a First Thoracic Vertebra and the Force Input Plus the Model's Predicted Acceleration

the frequency domain, partial differential equations also can be transformed into algebraic equations in the frequency domain. Each lumped parameter idealization characterizes mass with a single parameter. Continuous media idealizations, on the other hand, characterize mass by five parameters as indicated in Figure 1.

A Simple Continuous Media Idealization of the Impact Response of the Thorax

The final stages in utilization of the Inverse Method are to determine an idealization or model as well as the parameters associated with that model. Trauma producing blunt thoracic impacts can be further studied analytically using models of the thoracic system's impact response. There have been several mathematical models of the human thorax [See 5-8].

When using the mechanical impedance procedure to model the impact response of the human thorax, one common technique is to determine the parameters for a model only from the mechanical impedance magnitude. However, relying solely on the mechanical impedance magnitude will work in a very limited number of cases. Because transfer functions are complex-valued functions, it is necessary to consider both the real and imaginary values when constructing a model. This means that it is necessary to consider both the mechanical impedance magnitude and phase angle when constructing a model. For example, consider Figure 2 which represents both the magnitude and phase angle of the mechanical impedance transfer function. If one were to rely solely on the magnitude, a two mass model as shown in Figure 3 would seem reasonable. However, a two mass model, having mass, spring, and damper values for its parameters, least-squares fitted only to the mechanical impedance magnitude does not accurately reproduce the acceleration time-history from the laboratory experiment as shown in Figure 4. To generate output from the model, the inverse transform function in the frequency domain, representing the response of the thorax at a point, is obtained. This frequency domain function is created as a complex product of the Fourier Transform of the force and the appropriate transfer function representation of the model. To produce a better representation in the time domain, it is necessary to find the parameters for a model by using both the mechanical impedance magnitude and phase angle in a least squares approximation. The result of using both the mechanical impedance magnitude and the phase angle is shown in Figure 5. Inspection of the acceleration time-history indicates that, although, in a gross sense, this representation is in the ball park, some improvement can be made.

Inspection of the phase angle diagram of most UMTRI thoracic impacts of cadaver surrogates in conjunction with their respective auto- and cross-correlations, indicated that the most appropriate thoracic impact idealization would be a continuous media model. The continuous media model initially chosen is represented in Figure 6. The model in Figure 6 represents a cylindrical continuous media connected to a spring and damper in parallel, which is connected to a mass in series. The parameters for the model were determined by least-squares fitting the model to the experimental mechanical impedance transfer functions for the sternum, the right fourth rib, and the first thoracic vertebra, simultaneously. Although there is an improvement over the results shown in Figure 5, some aspects need to be improved in terms of the wave shape and phase angle. Figures 7, 8 and 9 show the results from the model. Figure 7 shows the acceleration time-history of the sternum and the force

from the actual experiment as well as the acceleration of the model. Figure 8 is similar and shows the acceleration of the right fourth rib for the actual data and the acceleration of the model. Likewise Figure 9 shows the actual data and the acceleration of the first thoracic vertebra. From the analysis of this output, the model pictured in Figure 10 was constructed. Figure 10 represents a model in which the soft-tissue has been separated from the hard tissue and in which the ribs have been broken up into four groups of three ribs each. In addition, the sternum and thoracic spine have been divided into two pieces. This model is considerably more complex than the one shown in Figure 6, and may be more complex than it needs to be for maximum efficiency. Figures 11, 12, and 13 show the output from the newly revised model. Figure 11 shows the acceleration time-history of the sternum and the force from the actual experiment as well as the acceleration of the model. Figure 12 is similar and shows the acceleration of the right fourth rib for the actual data and the acceleration of the model. Likewise Figure 13 shows the actual data and the acceleration of the first thoracic vertebra. The results indicate that a much better fit for a given test can be obtained through the use of the **Inverse Method**. However, this example has been part of an attempt to develop tools and the necessary procedures to utilize the **Inverse Method** for biomechanics experiments, and considerable work still needs to be done before the technique proves useful in biomechanics in the "wide sense." At this point in time, the following observations are pertinent:

- 1) The least-squares fitting routine developed at UMTRI for fitting mechanical impedance transfer functions to a given model is still in its infancy. Potentially, it may have been possible to obtain a better fit for the simple model presented in Figure 6 with a better program.
- 2) Although the model presented in Figure 10 produces a fairly good fit, it does not necessarily mean that the parameters determined from the least-squares fit are realistic. A procedure needs to be developed for evaluating the quality of the predicted parameter.
- 3) Only a very limited number of experiments were used to generate the models. A procedure for determining the necessary parameters to fit a number of experiments simultaneously in the least-squares sense needs to be developed.

The following are important advantages of this UMTRI modeling procedure:

- 1) A number of models can be quickly evaluated as to their ability to match experimental data.
- 2) It is relatively simple to generate more complex models for both lumped-parameter and continuous media using this technique.
- 3) No matrix inversions are needed. The technique used the Fast Fourier Transformation primarily.
- 4) The acceleration at any given point on a continuous media can be calculated.

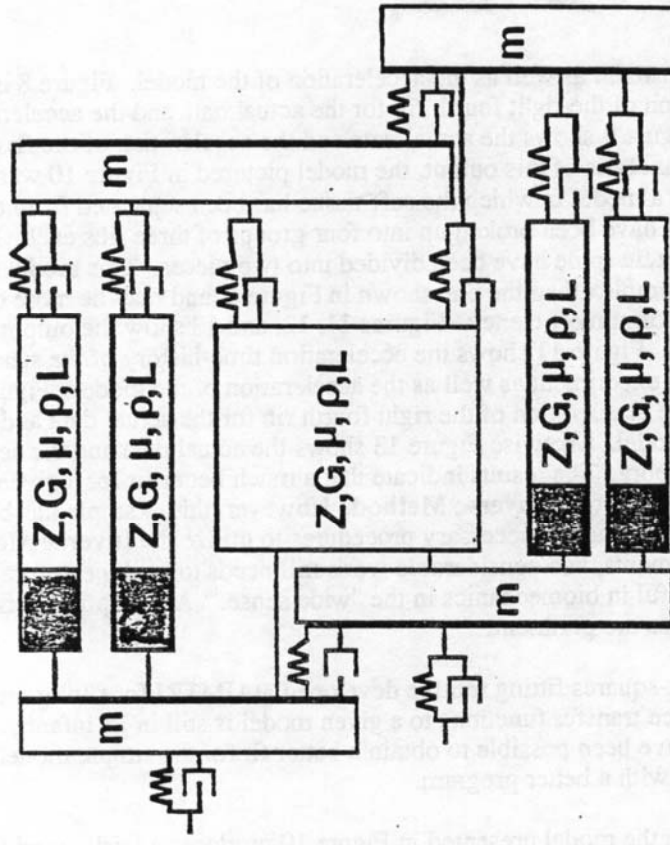
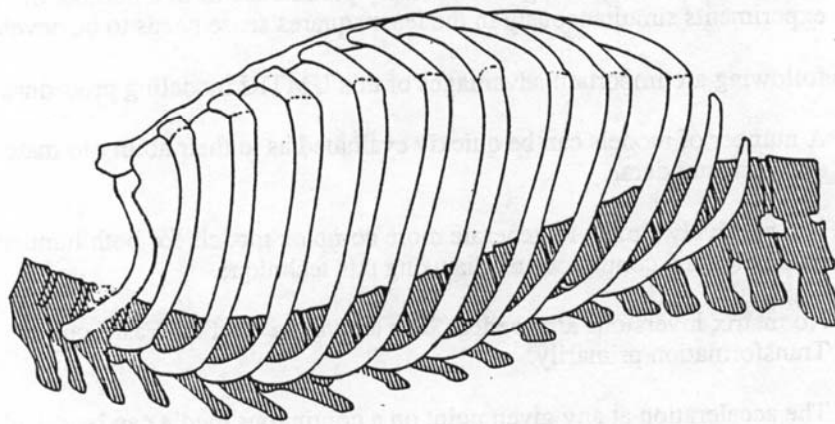


Figure 10. A More Complex Continuous Media Model of the Thorax

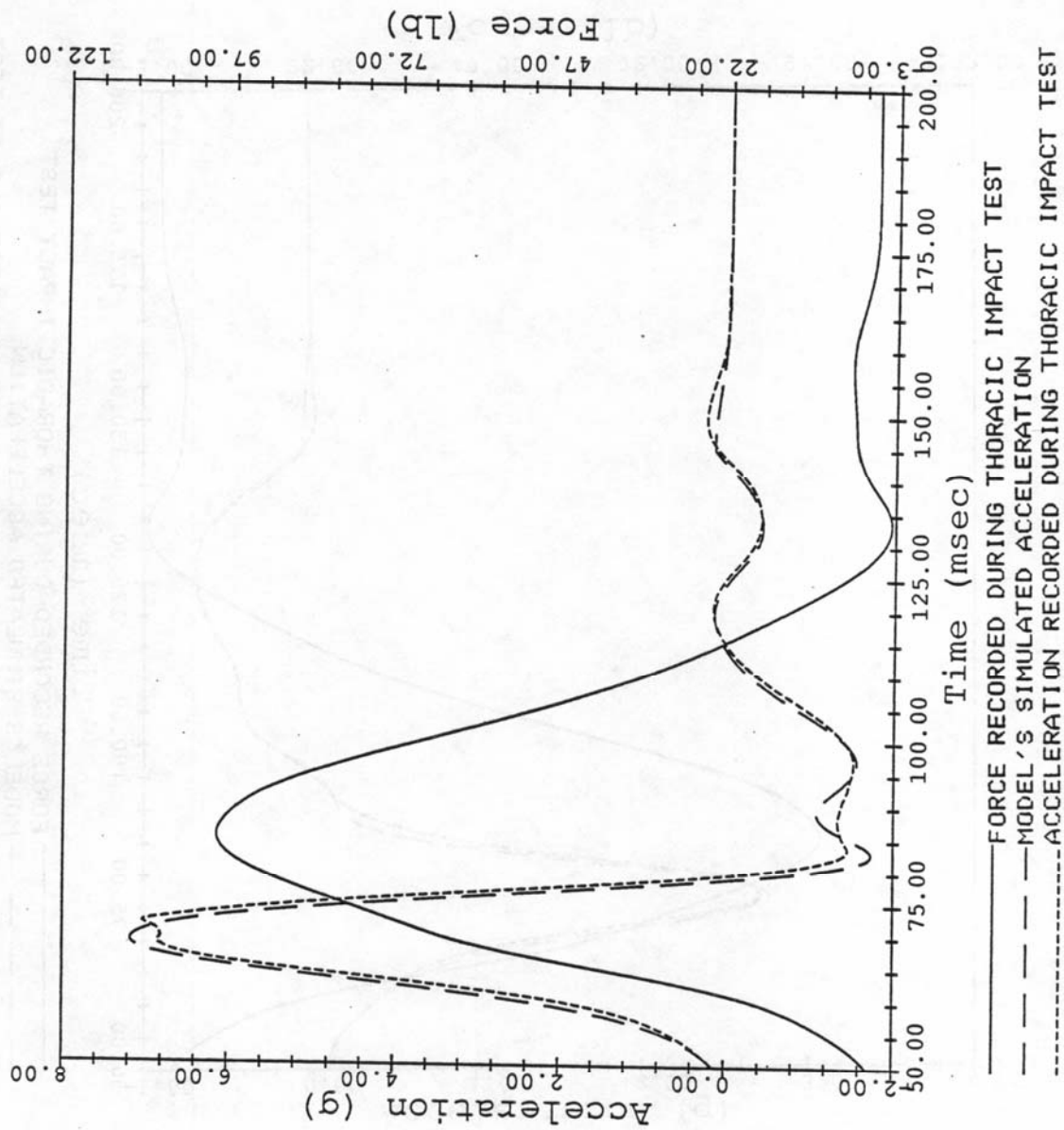


Figure 11. Comparison of an Acceleration Time-History of a Sternum and the Force Input Plus the Model's Predicted Acceleration

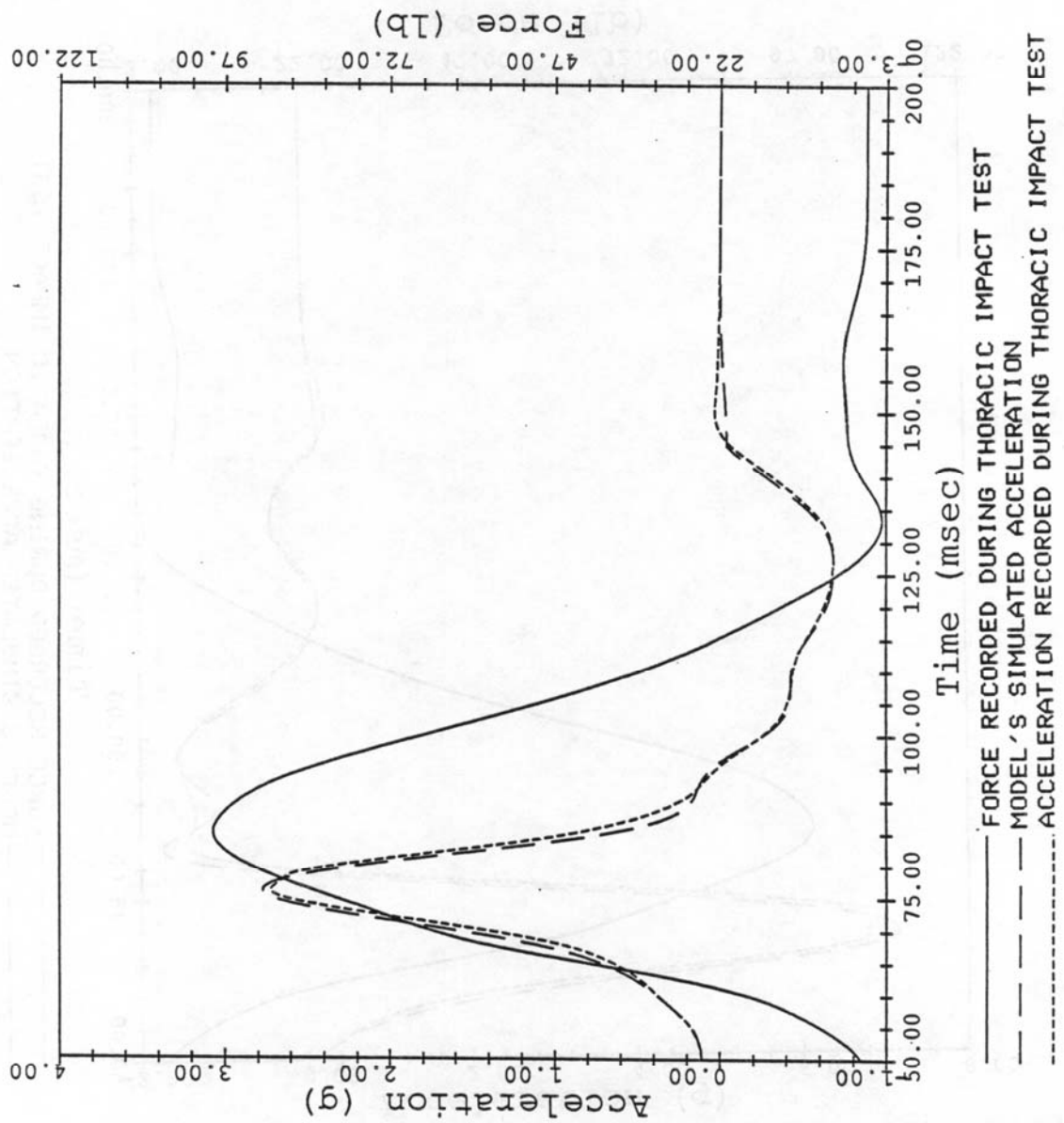


Figure 12. Comparison of an Acceleration Time-History of a Right Fourth Rib and the Force Input Plus the Model's Predicted Acceleration

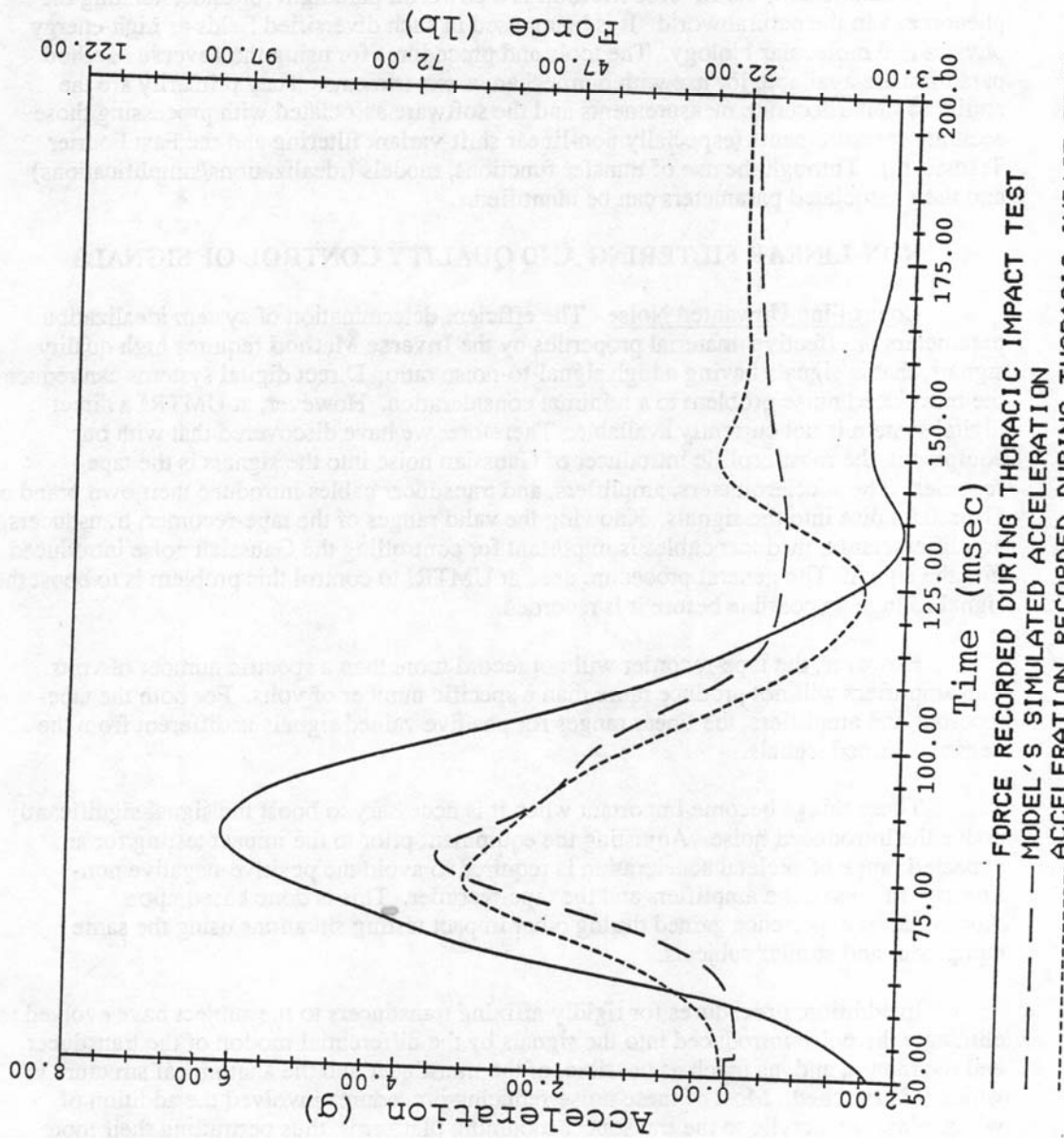


Figure 13. Comparison of an Acceleration Time-History of a First Thoracic Vertebra and the Force Input Plus the Model's Acceleration Recorded During Thoracic Impact Test

- 5) It is not necessary to calculate all points in the continuous media, which is unlike the finite-difference and finite-element approaches. A single point of interest can be calculated.

In conclusion, the **Inverse Method** is a powerful paradigm for understanding the phenomena in the natural world. It is being used in such diversified fields as high-energy physics and molecular biology. The tools and procedures for using the **Inverse Method** paradigm are available for use with biomechanics experiments. They primarily are the ability to make accurate measurements and the software associated with processing those accurate measurements (especially non-linear shift-variant filtering and the Fast Fourier Transform). Through the use of transfer functions, models (idealizations/simplifications) and their associated parameters can be identified.

NON-LINEAR FILTERING AND QUALITY CONTROL OF SIGNALS

Controlling Unwanted Noise - The efficient determination of system idealization parameters or effective material properties by the **Inverse Method** requires high quality signals, that is signals having a high signal-to-noise ratio. Direct digital systems can reduce the introduced noise problem to a minimal consideration. However, at UMTRI a direct digital system is not currently available. Therefore, we have discovered that with our equipment, the most prolific introducer of Gaussian noise into the signals is the tape-recorder. The accelerometers, amplifiers, and transducer cables introduce their own brand of Gaussian noise into the signals. Knowing the valid ranges of the tape-recorder, transducers, amplifiers, and transducer cables is important for controlling the Gaussian noise introduced into the signal. The general procedure used at UMTRI to control this problem is to boost the signal as high as possible before it is recorded.

However, the tape-recorder will not record more than a specific number of volts. The amplifiers will not produce more than a specific number of volts. For both the tape-recorder and amplifiers, the linear ranges for positive valued signals is different from the negative valued signals.

These things become important when it is necessary to boost the signal significantly above the introduced noise. Adjusting the equipment prior to the impact testing for an expected range of skeletal acceleration is required to avoid the positive-negative non-linearity ranges of the amplifiers and the tape-recorder. This is done based upon biomechanist experience gained during other impact testing situations using the same equipment and similar subjects.

In addition, procedures for rigidly affixing transducers to the subject have evolved to eliminate the noise introduced into the signals by the differential motion of the transducer and the mount, and, as much as possible, of the transducer and the anatomical structure to which it is attached. Most of these noise-reducing procedures involved the addition of wires, pins, and acrylic to the transducer mounting platforms, thus permitting their more rigid attachment to the anatomical structure. After these laboratory routines have helped to

produce raw data signals with the least amount of extraneous noise possible given our experimental situation, the recorded signals are next pre-processed to further increase the signal-to-noise ratio using a non-linear shift-variant filter.

UMTRI Non-Linear Shift-Variant Digital Filter - All transfer function techniques are plagued with the common problem of detecting the presence of a particular type or class of signal in the presence of noise. How do we process the data samples to obtain the best estimates of the signal with minimum noise? A common practice is to use the mean as an estimate of the true value of the signal. In the case of a time-varying signal, the practice is to sum several signals which are believed to be identical, after each signal has been properly aligned. Each point is divided by the number of summation. If the noise is Gaussian, as the number of samples goes up, error decreases as the square root of the number of samples. However, for the case of the short-lived non-repeatable phenomena occurring in blunt impacts to a human surrogate, this technique is not applicable.

In general, when time-histories are transformed into the frequency domain, both the signal and the noise contribute in some degree to the magnitude of each Fourier component. Low pass filters are particularly useful when the low frequency components of the Fourier transform of the transducer time-history are primarily a result of the signal and the high frequency components are primarily a result of noise. Low pass linear filters such as the Butterworth attenuate the high frequency components of the Fourier transform of the transducer time-history in a prescribed manner. However, the Butterworth or linear filters do not change the relative contribution of the signal and the noise to each Fourier component. One means of improving the signal-to-noise ratio is through operations performed in the experimental laboratory. However, this is not always practical. Another possibility is to use a preprocessor like the non-linear shift-variant filter being described in this paper. That filter does change the relative contribution of the signal and the noise to each Fourier component. It works by operating first in the time domain such that when a Fourier transform is performed on the transducer time-history, the relative contribution of the signal to each Fourier component is improved. This is primarily an advantage for the frequency domain when a transfer function between two transducer time-histories is generated for analytical purposes, although some improvement for the time domain is also achieved.

There are two aspects associated with the implementation of this non-linear shift-variant filter: 1) characterization of the Gaussian noise, and 2) estimation of the time-history using a power series. For most biomechanics impact experiments, the system under study can be viewed as causal or non-anticipatory. A causal system is one whose output does not depend on future values of the input. A noncausal system is one for which this condition is not assured. Noncausal systems do not, in general, exist in the macrophysical world, but are an important consideration in quantum mechanics and in some signal processing, such as digital filtering. A causal system has the property that only the values of the input and output of the system, at a given time and prior to that given time, affect the value at that given time. No values subsequent to the given time are required in order to compute the output of that system.

Considering the impact of a free-moving mass into a biological structure, the input (the impactor force) and output (the acceleration of the biological structure) responses (transducer time-histories) of a biodynamic system can be viewed as causal, provided that they were not filtered with a noncausal filter. In our practice, this is done by not filtering beyond the anti-aliasing filtering done before digitization of the time-histories. A period of time before impact (time zero) is recorded and digitized. For this period of time, it is assumed that fluctuation in the signal is a result of random or Gaussian noise with no aspect related to the behavioral response of interest. The noise observed in the "time zero" interval of the signal is assumed to be similar to the noise seen after that interval. Further, it is assumed that the mean of these observed values is a valid estimate of the value during the pre-impact interval. The mean can be subtracted from the rest of the signal to remove certain aspects of very low frequency noise from the entire time-history. The variance σ^2 associated with this mean can be used in removing some of the higher frequency noise.

Any transducer time-history consists of a true signal and noise. Therefore, given any discrete transducer time-history $u(t)$ of m points with error, we can choose any subset of that time-history $s(t)$ of j adjacent points for which j is less than m , so that:

$$s(t) = s_i(t) + \epsilon \quad (13)$$

where $s_i(t)$ is the true uncorrupted time-history, and ϵ is the noise associated with the time-history. We fit (i.e., approximate) this subset by a function $x(t)$ obtained from a polynomial of degree n , where $j > n$:

$$x(t_i) = \sum_{k=0}^n C_k t_i^k \quad (14)$$

Where $x(t_i)$ represents an approximation of a point on the time-history $s(t)$, and C_k is the least squares polynomial coefficient, $x(t)$ is the approximation of $s(t)$. In general, we cannot hope to find coefficients so that the polynomial approximation exactly fits the time-history $s(t)$.

Since we assumed that all the measurements were made at the correct time, all the errors are in the magnitude of the time-history $s(t)$. The mismatches associated with the polynomial fit of the time-history $s(t)$ are the "residuals." The magnitude of the variance associated with the residuals will be:

$$\sigma^2 = \frac{\sum_{i=1}^j (s(t_i) - x(t_i))^2}{j} \quad (15)$$

The variance σ^2 will be dependent upon the characteristics of the error (i.e. noise) and upon how close the true time-history $s_i(t)$ is to the approximation of $s(t)$ by a polynomial of degree k .

Polynomial fitting of a collection of time-histories $s(t)$ that span the time-history $u(t)$ can be an effective means of filtering $u(t)$. This is done by using a moving average filter or

convolution window. If all overlapping time-histories of a set number of adjacent points are least squares fitted with a fixed degree polynomial, then the collection of approximations can be used to construct a low pass filter. The number of points in the convolution window as well as the degree of the polynomial determine the characteristics (e.g. roll-off, stop-ban, pass-ban) of the filter. When using a low pass filter such as the Butterworth, the elliptic, the Chebychev, or one formed from polynomials, the assumption that is being made is that the frequency content of the true signal $u(t)$ is of much lower frequency content than that of the noise. When the frequency content of the signal spans the same range as the noise, then any reduction in the noise accomplished by using a linear filter is associated with a corresponding reduction in the signal. In linear filtering, a set of characteristics for the filter are defined for the entire time-history $u(t)$.

For dynamic biomechanics experiments, we want to filter different segments $s(t)$ of the time-history $u(t)$ differently. Figure 14 is a set of time-histories from a dynamic blunt impact experiment. As can be seen, before impact (i.e. during "time zero") there is only a DC component associated with the true uncorrupted signal. At impact, there is a sharp rise (i.e. a high frequency aspect of the true signal) with lower frequency aspects of the signal occurring after impact. We would like to pass only the lowest frequency components of the pre-impact segment of the signal, and to pass the much higher frequency components of the rise segment of the signal. For the fall-and-ending segment of the signal, we would like to pass the frequency components which lie somewhere in between those of the other two segments of the signal. To do that we need a procedure for choosing which segment's $s(t)$ are approximated as well as the degree of the polynomial for each $s(t)$. The variance σ^2 is used for this purpose.

The procedure for choosing which $s(t)$ as well as the degree of the polynomial for each $s(t)$ is determined in the following manner. Consider that the residuals are a function of the noise and the degree to which the true curve fits the polynomial.

$$\epsilon_r = \epsilon + \epsilon_p \quad (16)$$

where ϵ_r is the residual, ϵ is the noise associated with σ^2 and ϵ_p is the error associated with the mismatch of the true signal to the polynomial approximation. If the noise is uncorrelated, then for large segments $s(t)$:

$$\begin{aligned} E(\epsilon_m, \epsilon_n) &= \sigma^2 \text{ if } m = n \\ E(\epsilon_m, \epsilon_n) &= 0 \text{ if } m \neq n \end{aligned} \quad (17)$$

where $E(\epsilon_m, \epsilon_n)$ is the expectation value, and, in general:

$$\sigma^2 \leq \sigma_r^2 \quad (18)$$

Because both the degree to which the true signal deviates from a polynomial together with noise contributes to the magnitude of σ_r^2 , it is reasonable to assume that the polynomial fit $x(t)$ when $\sigma_r^2 \leq \sigma^2$ is as good an approximation of the true time-history as the original signal. Therefore, for any given $s(t)$ with j points for which:

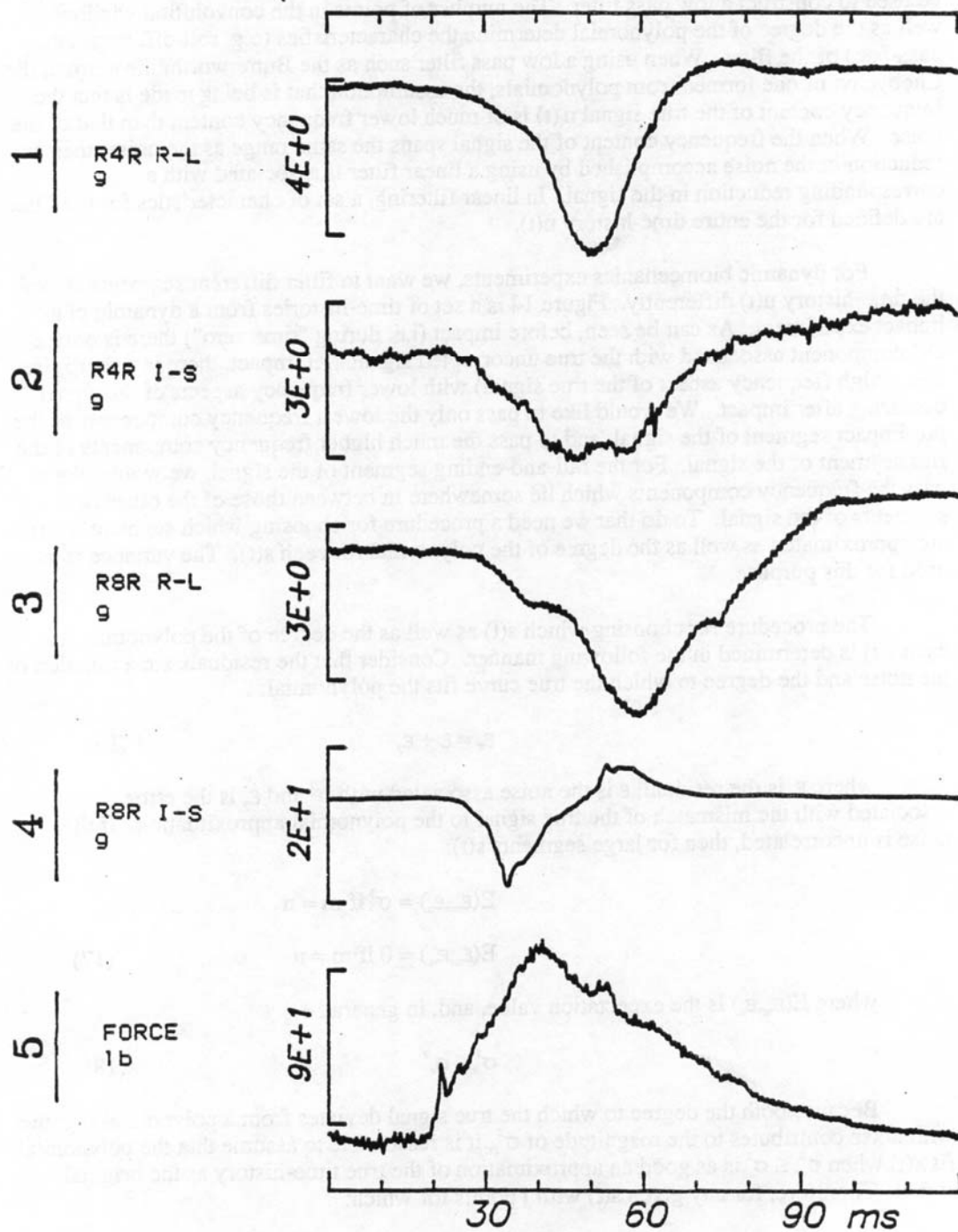


Figure 14. Acceleration time-history for a thoracic impact. R4R R-L is the acceleration response of the right fourth rib in the right to left direction. R4R I-S is the response of the right fourth rib in the inferior-superior direction. R8R is the response of the right eighth right rib in the right to left direction. R8L is the response of the left eighth rib in the right to left direction.

$$\sigma^2 \leq \sigma_r^2 = \frac{\sum_{j=1}^j (s(t_j) - x(t_j))^2}{j} \quad (19)$$

we replace the entire $s(t)$ with $x(t)$ to produce the filtered data. $x(t)$ now represents a filtered signal in which some of the high frequency noise has been removed. However, a given point $s(t_i)$ on the time-history $u(t)$ may be included in a number of $s(t)$'s and may be approximated a number of times. Therefore, let: $s(t)_i$ be the set of all $s(t)$ that include $s(t_i)$ with $\sigma_r^2 \leq \sigma^2$ and:

$$x(t_i) = \sum_{k=1}^{k_{\min}} C_k t_i^k \quad (20)$$

where $x(t_i)$ is the l th approximation of $x(t)$ for a given element of $s(t)_i$ and \min is the lowest order polynomial in which $\sigma_r^2 \leq \sigma^2$. A filter signal $u_f(t)$ can be constructed as:

$$u_f(t_i) = \frac{\sum_{k=1}^{k_{\min}} X(t_i^k)}{1} \quad (21)$$

where $u_f(t_i)$ is an element of the filtered signal and $u_f(t)$ represents the filtered signal. This result has the effect of filtering the different segments $s(t)$ of $u(t)$ differently. Therefore the condition $\sigma_r^2 < \sigma^2$ controls which $s(t)$ are approximated and which order polynomial is chosen. In those segments in which the signal is slow in varying, the filter tends to remove more higher frequency components than it does for those segments of the signal that are rapidly varying. It is believed that the resulting signal $u_f(t)$ is a better representation of the true signal $u(t)$ than $u(t)$ and that the usable high frequency range of the transfer function $x(i\omega)$ formed between two time-histories is increased.

The following factors are important for the implementation of the filter:

1. For any given $s(t)$, the polynomial chosen is the lowest order in which $\sigma_r^2 \leq \sigma^2$.
2. It is necessary to check all $s(t)$ of adjacent points. However, it is desirable to limit the maximum number of points in a given $s(t)$; the limit is chosen on a basis of limited computer time.
3. It is necessary to limit the maximum degree of the polynomial. This is a result of the fact that the condition $\sigma_r^2 \leq \sigma^2$ may not exist for a given $s(t)$.
4. All points on $u(t)$ may not be included in at least one reconstruction $x(t)$. In that case, the original point $u(t_i)$ is placed into the filtered signal.

In summary, the following four aspects of this filter may differ from other filters:

1. In the common case of smoothing, the midpoint of a segment $x(t)$ is used to replace the original point $s(t_m)$ with $x(t_m)$, this filter replaces all $s(t)$ with $x(t)$;

2. The filter can be self-starting and ending because of the aspect just mentioned;
3. The number of points in the window, as well as the degree of polynomial, are different for different segments $s(t)$;
4. This is a low pass filter in which the filter characteristics differ for different segments $s(t)$;
5. It is necessary to have a segment of the signal in which the true signal $s_i(t)$ is known in order to determine σ^2 .

This filter is similar in nature to a moving average filter, except that the degree of the filter polynomial is determined by the values of the points being filtered, as well as by the number of points. In this regard, the filter is shift-variant. In addition, if two signals are added, the noise before time zero changes and the filter will have a different effect on the time-history of the summed signals, than it would have on the effect of filtering the signals first and then summing them. Thus, the filter is nonlinear. The filter has been used repeatedly at UMTRI on over 10,000 different time-histories with very successful results. It has shown itself to be conservative. However, the filter does have one drawback; the current algorithm is slow and should be speeded up. This revision is currently in progress.

Example of How This Filter Affects Kinematic Signals

A signal that represents the contact force time-history generated when a 25 kg mass strikes the side (i.e., the thorax) of a repressurized cadaver was created. The signal was constructed from the equation:

$$F(t) = C_1 \cdot \exp(C_2(C_3-t)) \cdot \operatorname{sech}(C_4(C_5-t)) \quad (22)$$

where $C_1 \dots C_5$ are constants.

The constructed signal was least-squares fitted by adjusting the constants $C_1 \dots C_5$ to an actual time-history obtained from a biomechanics experiment conducted at UMTRI. This fitted signal was used as the unaltered time-history and is referred to as the "base signal." In the analysis of biomechanics experiments, kinematic variables are compared using a transfer function created from the two transducer time-histories (e.g., a force time-history and an acceleration time-history), it is assumed that each time-history has the same degree of noise. In the example presented here, both time-histories are processed identically with the filter being described in this paper. The effect on the transfer function of filtering just one time-history can have unacceptable results because the level and type of noise in each signal would be unknown, unlike the situation illustrated in this example. Figure 15 illustrates a transfer function of the base signal with itself to illustrate what the true transfer function should be. Figure 16 illustrates the time-history of the filtered base signal contrasted with the base signal having several different types of noise added as well as with these noisy signals run through the filter.

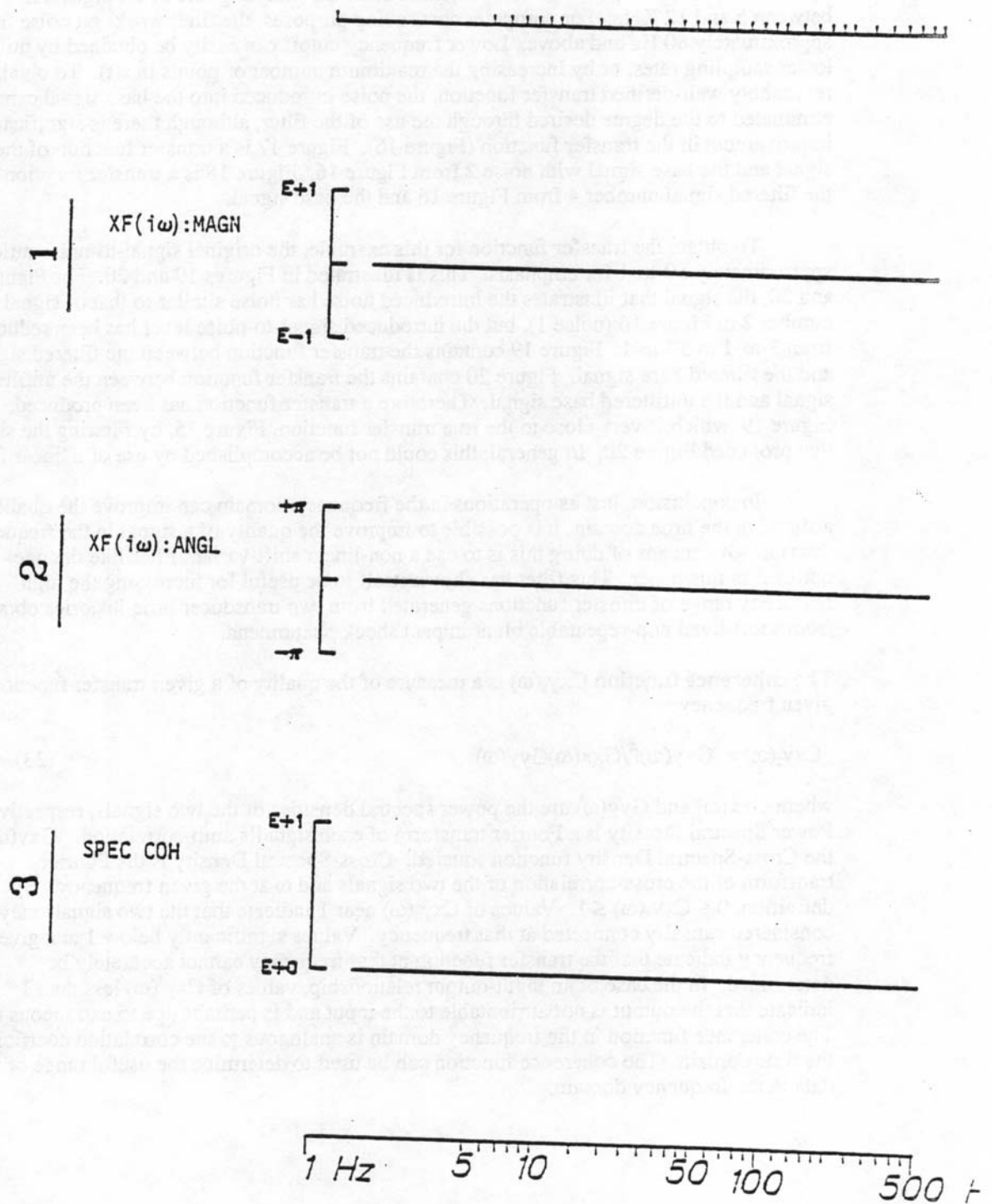


Figure 15. Transfer function of an artificial signal and itself. Figure 15 includes transfer function magnitude (XF:MAGN), transfer function angle (XF:ANGL), and spectral coherence for the signal with itself (SPEC COH).

In the filter implementation presented here, the sampling rate of the signals is between 5 and 12 KHz. For computer cost saving purposes, the filter works on noise from approximately 80 Hz and above. Lower frequency cutoff can easily be obtained by utilizing lower sampling rates, or by increasing the maximum number of points in $s(t)$. To obtain a reasonably well-defined transfer function, the noise introduced into the base signal cannot be eliminated to the degree desired through the use of the filter, although there is significant improvement in the transfer function (Figure 16). Figure 17 is a transfer function of the base signal and the base signal with noise 2 from Figure 16. Figure 18 is a transfer function of the filtered signal number 4 from Figure 16 and the base signal.

To obtain the transfer function for this example, the original signal-to-noise ratio was approximately 50-to-1 for emphasis. This is illustrated in Figures 19 and 20. For Figures 19 and 20, the signal that illustrates the introduced noise has noise similar to that of signal number 2 in Figure 16 (noise 1), but the introduced signal-to-noise level has been reduced from 3-to-1 to 50-to-1. Figure 19 contains the transfer function between the filtered signal and the filtered base signal. Figure 20 contains the transfer function between the unfiltered signal and the unfiltered base signal. Therefore a transfer function has been produced, Figure 19, which is very close to the true transfer function, Figure 15, by filtering the signals that produced Figure 20. In general, this could not be accomplished by use of a linear filter.

In conclusion, just as operations in the frequency domain can improve the quality of a signal in the time domain, it is possible to improve the quality of a signal in the frequency domain. One means of doing this is to use a non-linear shift-variant filter like the one outlined in this paper. This filter has shown itself to be useful for increasing the high frequency range of transfer functions generated from two transducer time-histories obtained from short-lived non-repeatable blunt impact shock phenomena.

The coherence function $C_{xy}(\omega)$ is a measure of the quality of a given transfer function at a given frequency:

$$C_{xy}(\omega) = |G_{xy}(\omega)|^2 / G_{xx}(\omega)G_{yy}(\omega) \quad (23)$$

where $G_{xx}(\omega)$ and $G_{yy}(\omega)$ are the power spectral densities of the two signals, respectively. Power Spectral Density is a Fourier transform of each signal's auto-correlation. $|G_{xy}(\omega)|^2$ is the Cross-Spectral Density function squared. Cross-Spectral Density is the Fourier transform of the cross-correlation of the two signals and ω at the given frequency. By definition, $0 \leq C_{xy}(\omega) \leq 1$. Values of $C_{xy}(\omega)$ near 1 indicate that the two signals may be considered causally connected at that frequency. Values significantly below 1 at a given frequency indicate that the transfer function at that frequency cannot accurately be determined. In the case of an input-output relationship, values of $C_{xy}(\omega)$ less than 1 indicate that the output is not attributable to the input and is perhaps due to extraneous noise. The coherence function in the frequency domain is analogous to the correlation coefficient in the time domain. The coherence function can be used to determine the useful range of the data in the frequency domain.

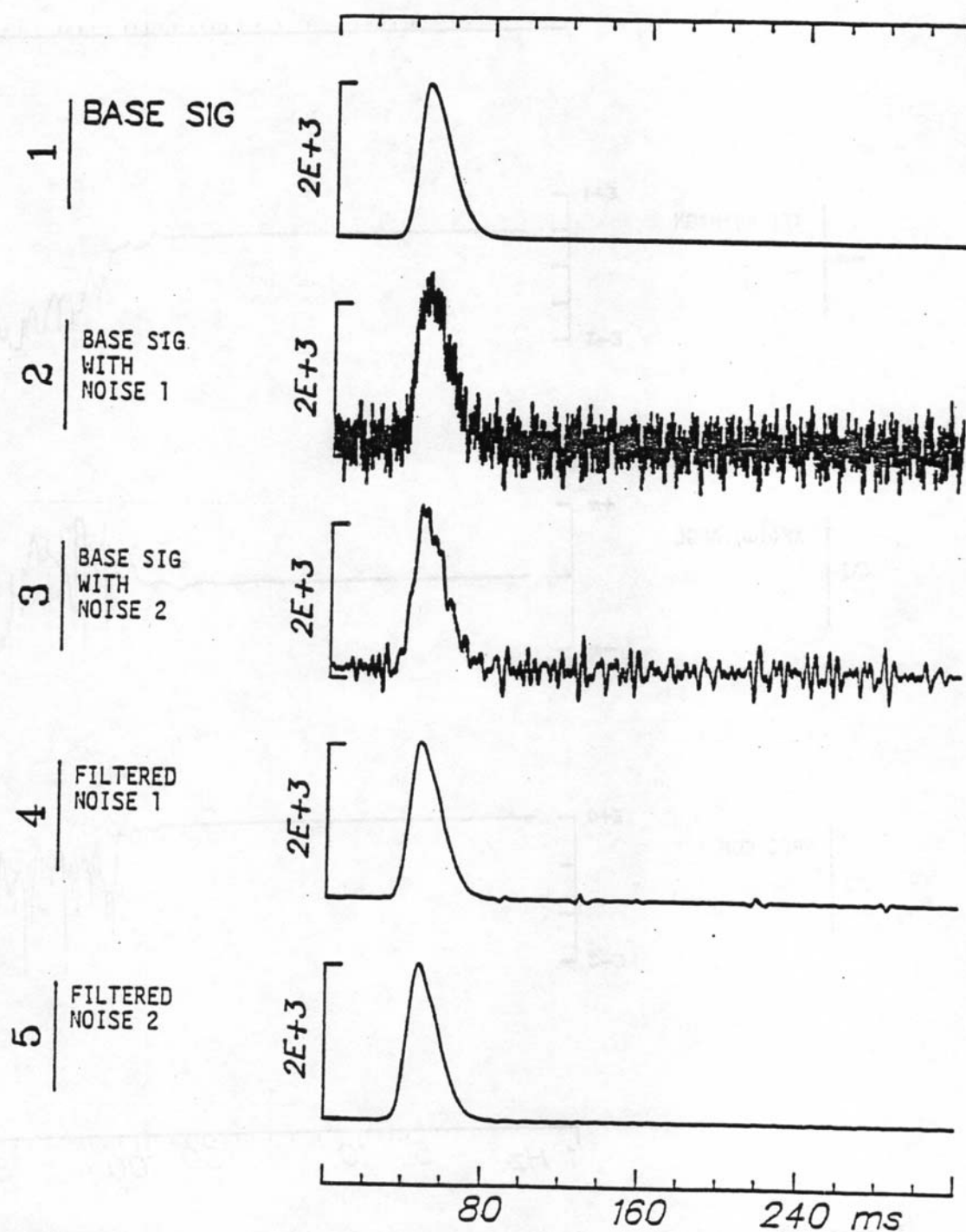


Figure 16. Artificial signal (BASE SIG - variable 1) with two types of noise; noise 1 and noise 2 (variables 2 and 3, respectively), filtered noise 1 and filtered noise 2 (variables 4 and 5, respectively).

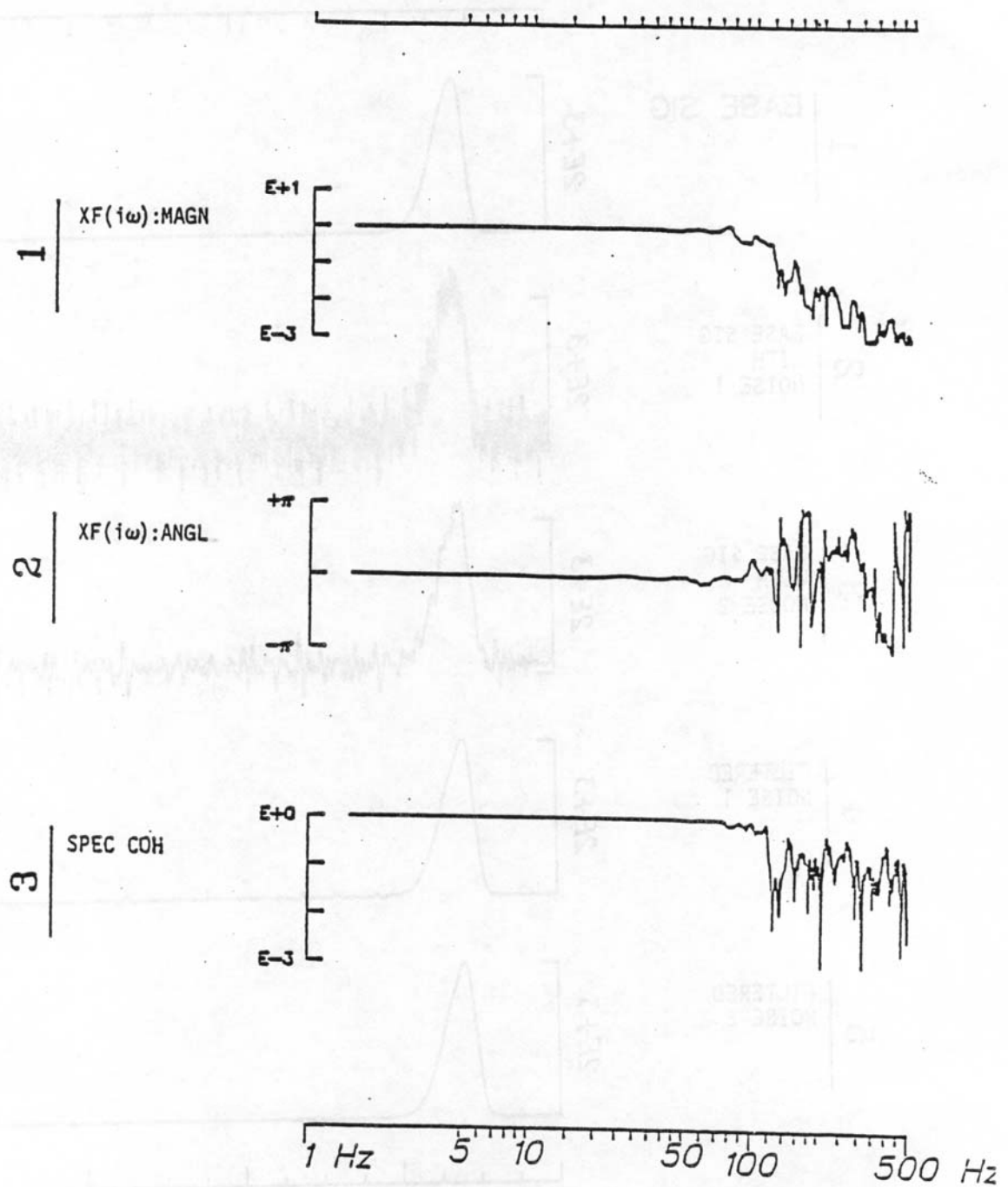


Figure 17. Transfer function for the base signal (variable 1 in Figure 16) and the base signal with noise 1 (variable 1 with variable 2 in Figure 16). Figure 17 includes transfer function magnitude (XF:MAGN), transfer function angle (XF:ANGL), and spectral coherence for the two signals (SPEC COH).

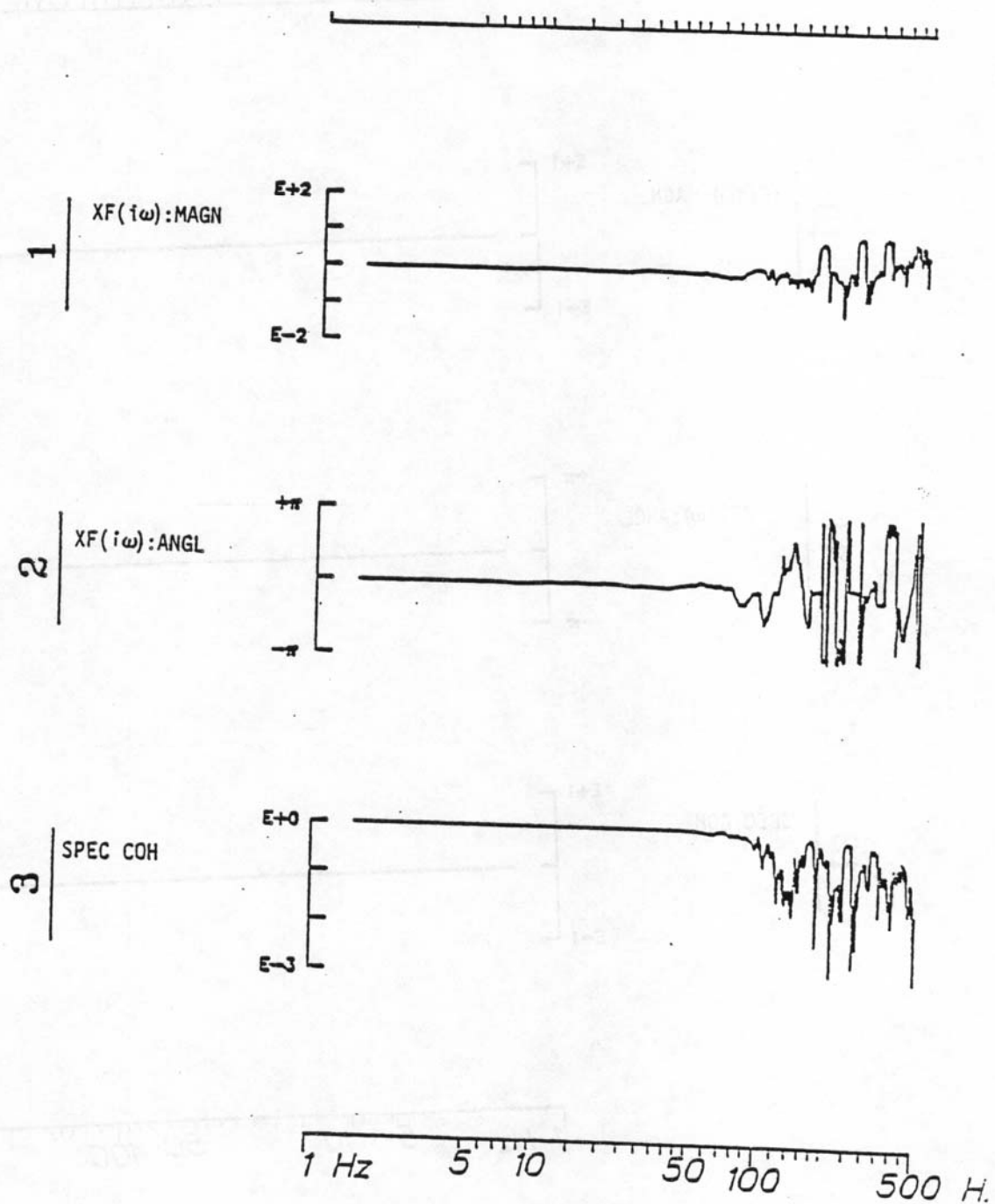


Figure 18. Transfer function for base signal (variable 1 in Figure 16) and the filtered base signal with noise 1 (variable 4 in Figure 16). Figure 18 includes transfer function magnitude (XF:MAGN), transfer function angle (XF:ANGL), and spectral coherence for the two signals (SPEC COH).

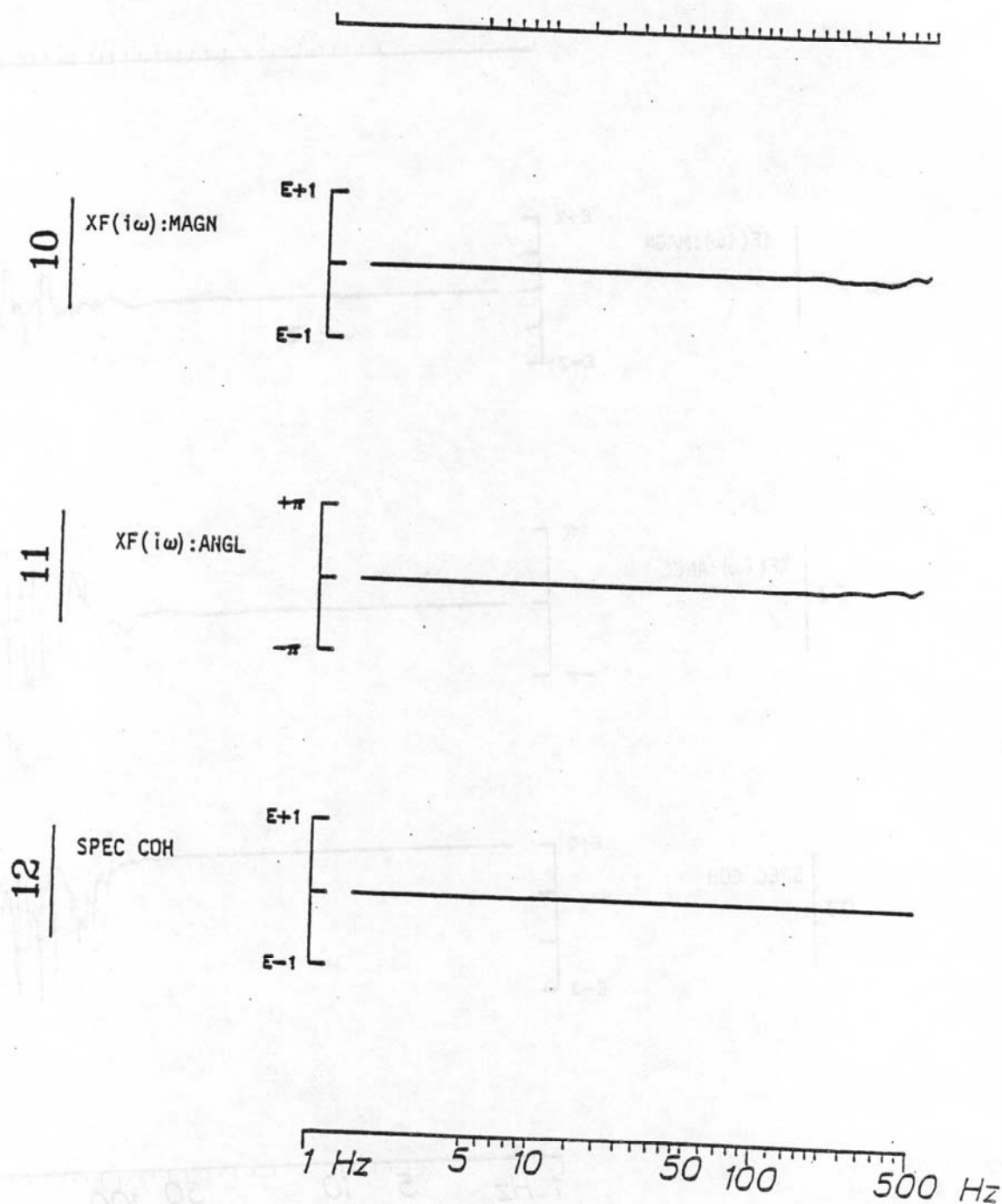


Figure 19. Transfer function for base signal and filtered base signal with noise 1 (variable 4 in Figure 16). However, the signal-to-noise ratio is reduced to 2.5% from 30%. Figure 19 includes transfer function magnitude (XF:ANGL), and spectral coherence for the signal with itself (SPEC COH).

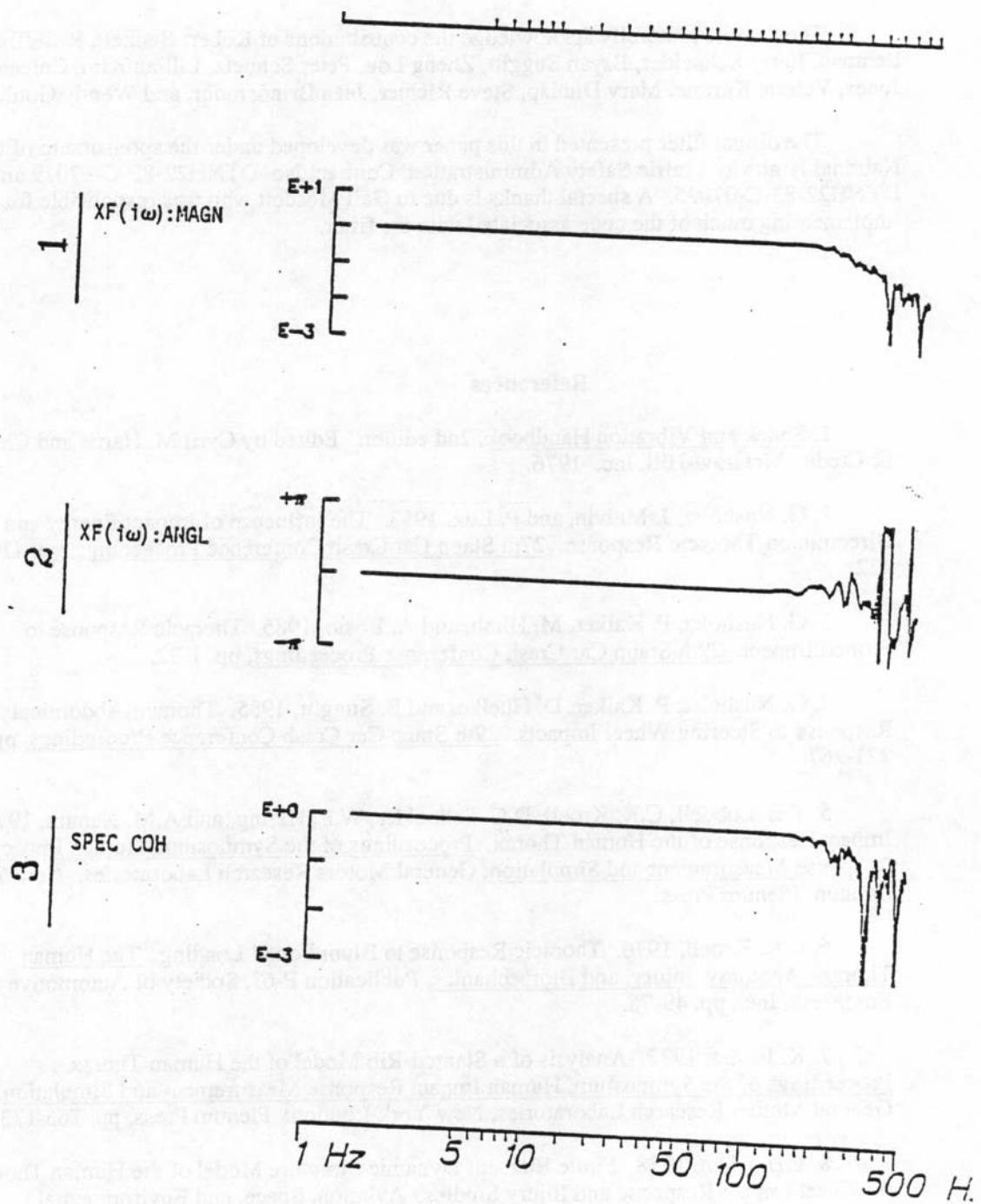


Figure 20. Transfer function for base signal and the unfiltered base signal with noise similar to noise 1 (variable 2 in Figure 16), but reduced so that the signal-to-noise ratio is 2.5% instead of 30%. Figure 20 includes transfer function magnitude (XF:MAGN), transfer function angle (XF:ANGL), and spectral coherence for the signal with itself (SPEC COH).

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DISCUSSION

PAPER: The Inverse Method, Transfer Function Analysis, and Non-Linear Filtering.

SPEAKER: G.S. Nusholtz

- Q: John Melvin, General Motors Research
You showed pretty good agreement in your last fit using a multi-multi parameter approach. With that many parameters certainly you can get a good fit. How predictive is it? Can you describe another test condition and get the right response without changing any parameters or how variable are the parameters that you find, say different test subjects? In other words is this just curve fitting or can you predict with this?
- A: With the increased number of parameters it makes it much easier to get the curve to fit and then when you invert the signal, you should if you've done the curve fitting correctly, get a very close agreement. Right now we've only done it on a limited number of tests, but it does seem to generally fit other impacts of that type. The problem we are having now is the thorax seems to be very velocity dependent, and this is a very linear model, If I change velocities of impact it doesn't fit that well, or if I go to other subjects and look at it, it's not quite as good as this. This is just a fit on one curve. That's how it came out.
- Q: But you do have a viscous term in there so it should be velocity dependent?
- A: Right, but it's even more. This is just an opinion, in my opinion it's even more velocity dependent than just that viscous term allows.
- Q: Jeff Marcus, NHTSA
The filtering that you do, I have two questions. First of all you don't actually have anything even remotely like a set frequency that you're filtering above and letting things pass through below, rather you're just trying to find the noise component regardless of what its frequency is and top the signal and get rid of that, is that correct?
- A: That's correct. There's no cut off. Some signals may cut off at 1000 Hertz, that start to roll off at 1000 Hertz. Others will roll off at maybe 100, it depends on what the noise properties are before impact. The filter is kind of fixed to an impact condition because you have to be able to define an area where you know there is noise and you know exactly that there's a true signal there, and you know what that true signal is. In this case a flat line.

Q: Before, when you had the slide showing how the filter worked, you compared two variances and said that if one variance was less than the other, then you knew something was true. But I wasn't sure the variance with respect to what?

A: Well, probably I'll have to talk to you after the presentation, but I'll try to make this short. There's two variances, one is you determine the variance from what you believe to be the true noise, then you determine the variance between your polynomial fit and the signal that you're looking at. If the variance of the polynomial fit is less than or equal to the variance of the true noise, then the polynomial fit gives you a good approximation of what the data are or a better approximation than just using the original data.